Practical Post-Quantum Signatures for Privacy

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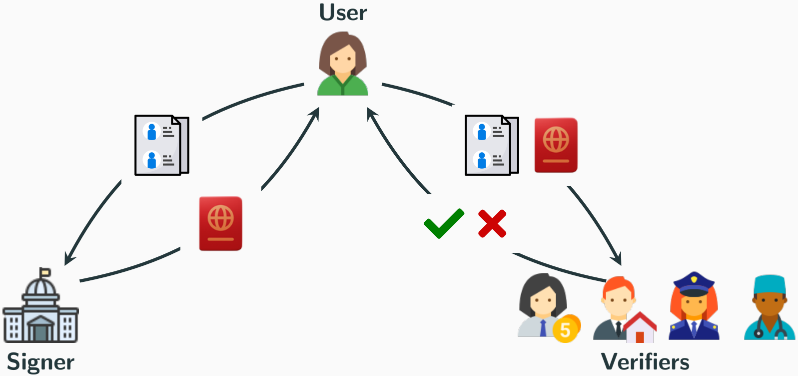
Joint work with Sven Argo, Tim Güneysu, Georg Land, Adeline Roux-Langlois, Olivier Sanders

Signatures: Physical and Digital



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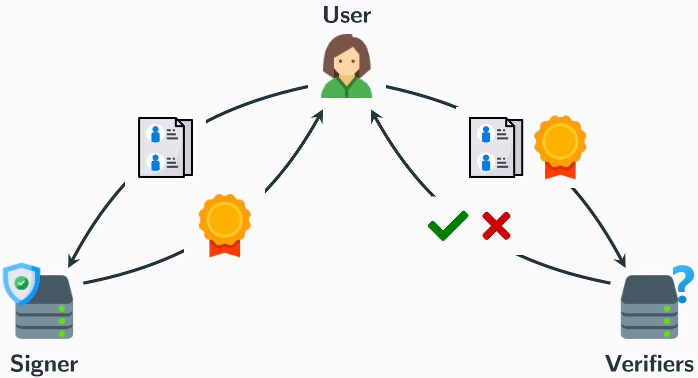
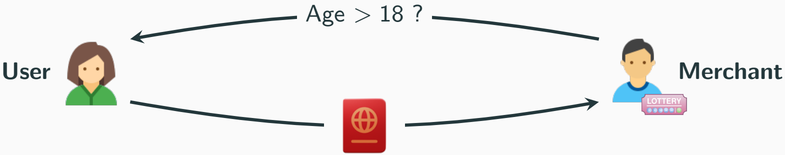


Allows to certify digital data, and later prove its authenticity. What more do we need?

Temporarily showing an ID document to attest you are of age is **not really a privacy issue**.



Example: Age Control

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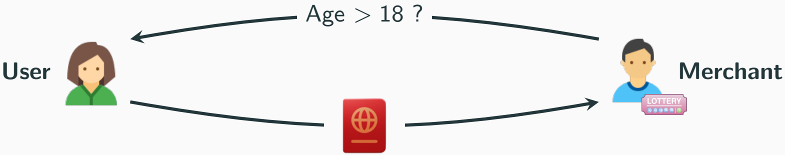
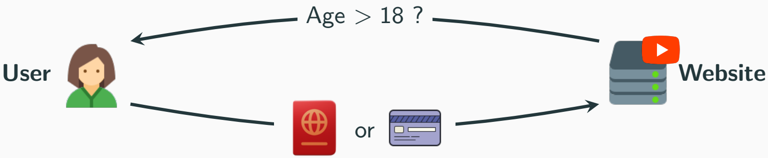


Sending an ID document or credit card to a website is more **permanent**. It can **store**, **share**, **exploit**. Requires **trust**.



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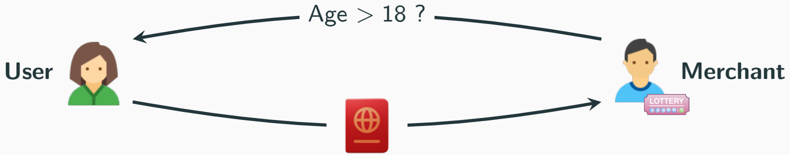

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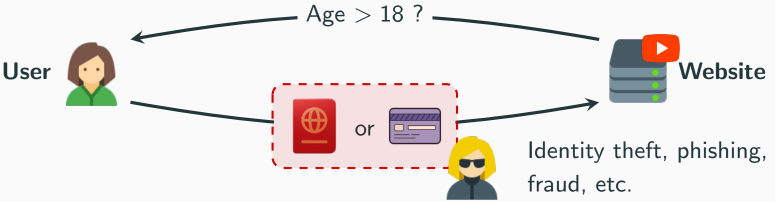
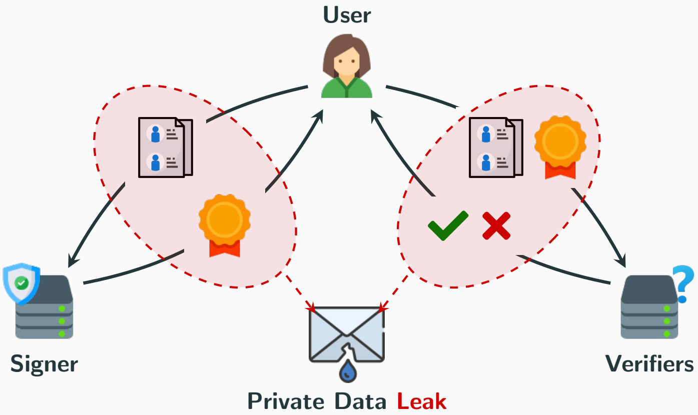
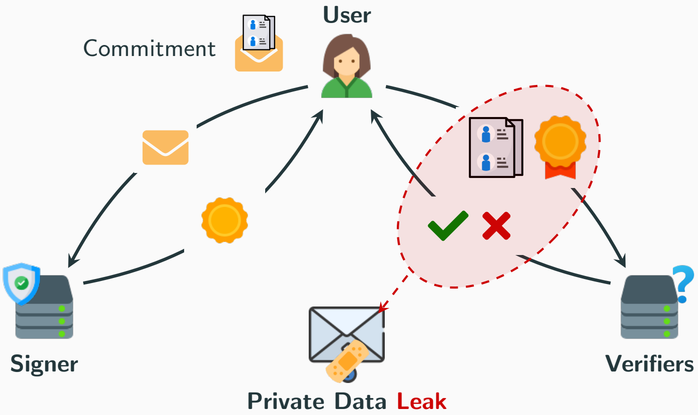




No control over the disclosed information: Verifiers (and attacker) learn everything

Simple but not suited for privacy



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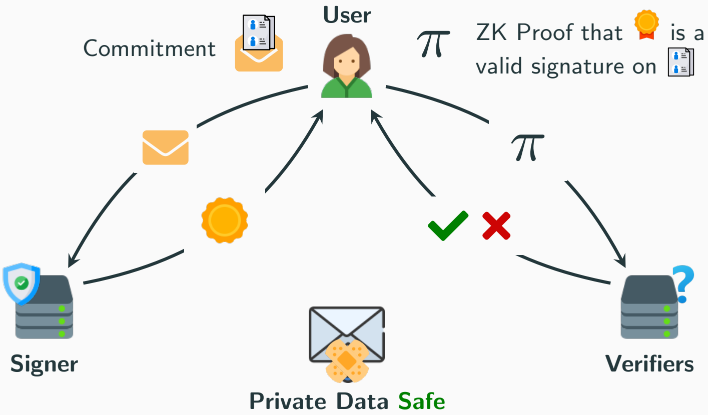
Adding Privacy: Signature with Efficient Protocols (SEP)



Full control of user information: Selective disclosure to verifiers (and attacker)
But need for more complex tools: commitment, specific signature, ZKP

An Interesting Versatility

Many technical solutions answering concrete privacy use cases can be built from this blueprint.

Anonymous Credentials · **Group Signatures** · **Blind Signatures** · **E-Cash** · …

All these need some **signature** with some kind of **anonymity**

Industrial Interest: EPID and DAA deployed in billions of devices (TPM, Intel SGX).
EPID, DAA, Group/Blind signatures in ISO/IEC standards (20008, 18370)

Most solutions **broken** **by Quantum Computers**.
Need **Post-Quantum** **alternatives**

Background

1. Lattices: Assumptions, Trapdoors & Samplers

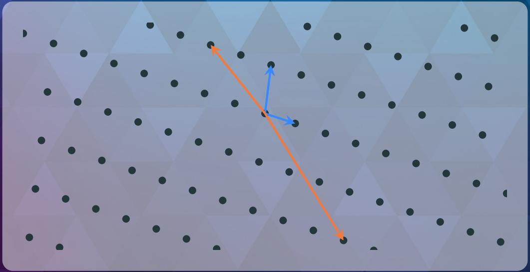
Construction

2. Signature with Efficient Protocols

Performance

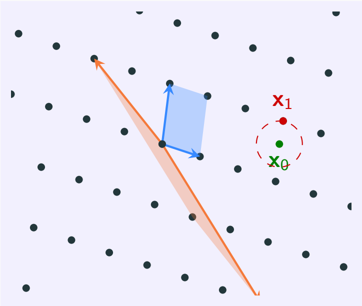
3. Anonymous Credentials Use-Case
Sizes & Timings

Lattices: Assumptions, Trapdoors & Samplers



Euclidean Lattice

$$\mathcal{L} = \left\{ \boxed{\mathbf{B}}\,\boxed{\mathbf{x}}\ ;\ \mathbf{x} \in \mathbb{Z}^n \right\} \text{ with basis } \mathbf{B} \in \mathbb{R}^{n \times n}$$



CVP$_{\mathbf{x}_0}$ Given a target $\mathbf{x}_0$, find $\mathbf{x}_1 \in \mathcal{L}$ that minimizes $\|\mathbf{x}_0 - \mathbf{x}_1\|$

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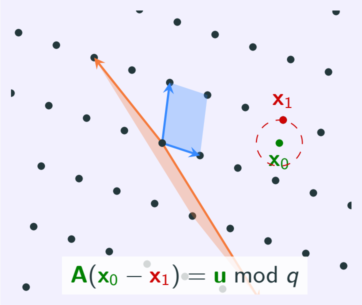

CVP$_{\mathbf{x}_0}$ Given a target $\mathbf{x}_0$, find $\mathbf{x}_1 \in \mathcal{L}$ that minimizes $\|\mathbf{x}_0 - \mathbf{x}_1\|$

Given $\mathbf{A} \in R_q^{d\times m}$ describing the lattice

$$\mathcal{L}_q^{\perp}(\mathbf{A}) = \{\mathbf{x}_1 \in R^m : \mathbf{A}\mathbf{x}_1 = \mathbf{0} \bmod q\}$$

and $\mathbf{x}_0$ such that $\mathbf{A}\mathbf{x}_0 = \mathbf{u} \bmod q$, solve **CVP$_{\mathbf{x}_0}$** on $\mathcal{L}_q^{\perp}(\mathbf{A})$. This is **ISIS**!

$\mathsf{ISIS}_{m,d,q,\beta}$

Given $(\mathbf{A}, \mathbf{u}) \hookleftarrow U(R_q^{d \times m+1})$, find $\mathbf{x} \in R^m$ such that $\mathbf{A}\mathbf{x} = \mathbf{u} \bmod q$, $\|\mathbf{x}\| \leq \beta$.
When $\mathbf{u} = \mathbf{0}$, we ask $\mathbf{x} \neq \mathbf{0}$.

Decision: Distinguish $\mathbf{A}\mathbf{x} \bmod q$ for a random short $\mathbf{x}$ from a random $\mathbf{u}$.

- Statistical Hardness — Leftover Hash Lemma
- Computational Hardness — Learning With Errors (LWE)

Lattice Assumption and Trapdoors

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ISIS is hard unless we know a trapdoor $\mathbf{R}$ on $\mathbf{A}$.

- Ability to invert $f_{\mathbf{A}} : \mathbf{x} \mapsto \mathbf{A}\mathbf{x} \bmod q$ over bounded domain
  - Ability to randomize preimage finding without leaking $\mathbf{R}$ → **Preimage Sampling**
    - Design secure signatures [GPV08][1]: Find short $\mathbf{x}$ such that $\mathbf{A}\mathbf{x} = \mathcal{H}(\mathbf{m}) \bmod q$

[1] Gentry, Peikert, Vaikuntanathan. Trapdoors for Hard Lattices and New Cryptographic Constructions. STOC 2008.

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Our main thread is **versatility**: Gadget-based Trapdoors [MP12][1]

[1] Micciancio, Peikert. Trapdoors for Lattices: Simpler, Tighter, Faster, Smaller. Eurocrypt 2012

Micciancio-Peikert trapdoors [MP12]: Family of matrices $\overline{\mathbf{A}}$ such that

$$\overline{\mathbf{A}}\mathbf{R}' = \mathbf{T}\mathbf{G} \bmod q, \quad \text{with} \quad \mathbf{R}' = \begin{bmatrix} \mathbf{R} \\ \mathbf{I} \end{bmatrix}, \quad \text{i.e.} \quad \overline{\mathbf{A}} = [\mathbf{A} | \mathbf{T}\mathbf{G} - \mathbf{A}\mathbf{R}] \text{ and } \mathbf{A} = [\mathbf{I} | \mathbf{A}']$$

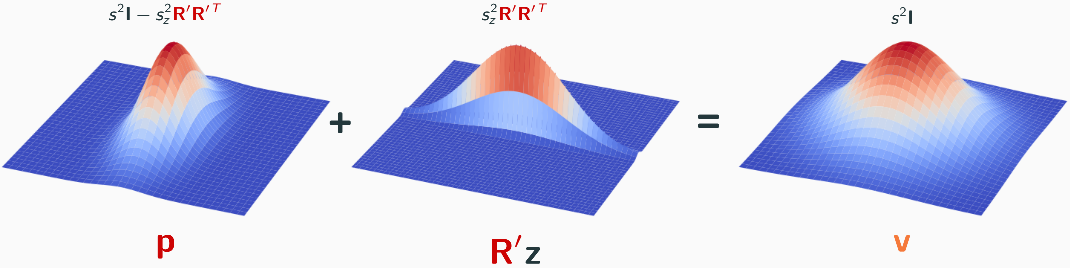
with $\mathbf{G} = \mathbf{I} \otimes [b^0 | \ldots | b^{k-1}]$, and $k = \log_b q$
(base-b decomposition)

$\mathbf{R}$ $\quad$ $\mathbf{B} = \mathbf{A}\mathbf{R}$
$\mathbf{T}$ $(= t\mathbf{I})$

Naive Approach: Compute $\mathbf{z}$ so that $\mathbf{T}\mathbf{G}\mathbf{z} = \mathbf{u} \bmod q$, and return $\mathbf{R}'\mathbf{z}$ as preimage of $\mathbf{u}$

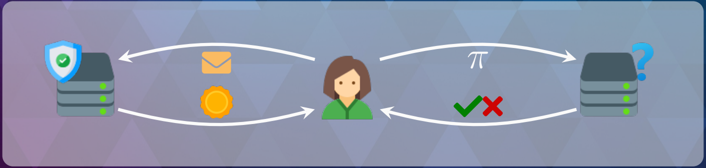
- Collecting many preimages will leak $\mathbf{R}$...
- Add mask $\mathbf{p}$: preimages $\mathbf{v} = \mathbf{p} + \mathbf{R}'\mathbf{z} = \begin{bmatrix} \mathbf{p}_1 + \mathbf{R}\mathbf{z} \\ \mathbf{p}_2 + \mathbf{z} \end{bmatrix}$ (and gadget inversion on $\mathbf{u} - \overline{\mathbf{A}}\mathbf{p}$ instead of $\mathbf{u}$)

Compensate statistical leakage by adapting covariance of $\mathbf{p}$ [MP12]. Only for $\mathbf{z}$ and $\mathbf{p}$ Gaussian



Quality: $s \gtrsim s_z\sqrt{1+\|\mathbf{R}\|_2^2}$ with $s_z \approx \eta_\varepsilon(\mathcal{L}_q^\perp(\mathbf{G}))$.

Lattice Signatures for Privacy: Versatile & Practical



Let's see if we can use **Falcon** to construct **Signatures with Efficient Protocols**

$$v_1 + h v_2 = \mathcal{H}(\mathbf{m})$$

✗ Need efficient ZKP of verification. Hash evaluation ($\mathcal{H}(\mathbf{m})$) is impractical to prove

Falcon/Dilithium with Efficient Protocols?

Same goes for **Dilithium** or **Micciancio-Peikert** signatures

: $\mathbf{R}$: $\mathbf{B} = \mathbf{A}\mathbf{R}$: $\mathbf{v}$: $\mathbf{m}$ PP : $(\mathbf{A}, \mathbf{G}_H = \mathbf{I} \otimes [b^\ell | \dots | b^{k-1}])$

$$\begin{bmatrix} \mathbf{A} & \mathbf{G} - \mathbf{B} \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ \vdots \end{bmatrix} = \mathcal{H}(\mathbf{m})$$

Need efficient ZKP of verification. Hash evaluation ($\mathcal{H}(\mathbf{m})$) is impractical to prove

Falcon/Dilithium with Efficient Protocols?

Where to put the message if not in the syndrome $\mathcal{H}(\mathbf{m})$?

$$\begin{bmatrix} \mathbf{A} & t(\mathbf{m})\mathbf{G} - \mathbf{B} \end{bmatrix} \begin{bmatrix} \mathbf{v} \\ \vdots \end{bmatrix} = \mathbf{u}$$

Tag function of the message [dPLS18][2] (group sig), [dPK22][3] (blind sig)

[2] del Pino, Lyubashevsky, Seiler. Lattice-Based Group Signatures and Zero-Knowledge Proofs of Automorphism Stability. CCS 2018

[3] del Pino, Katsumata. A New Framework For More Efficient Round-Optimal Lattice-Based (Partially) Blind Signature via Trapdoor Sampling. Crypto 2022

Where to put the message if not in the syndrome $\mathcal{H}(\mathbf{m})$?

$$\overline{\mathbf{A}}\,\mathbf{v} = \mathbf{u} + \mathbf{D} \cdots \cdot \mathrm{bin}\left(\mathbf{D}_0\,\mathbf{r} + \mathbf{D}_1 \cdots \mathbf{m}\right)$$

Commitment to the message using Chameleon hash [LLM+16][2]

[2] Libert, Ling, Mouhartem, Nguyen, Wang. Signature Schemes with Efficient Protocols and Dynamic Group Signatures from Lattice Assumptions. Asiacrypt 2016

Commitment, Convolution sampler, Elements t and $\mathbf{u}$ to prove security on SIS

: $\mathbf{R}$: $\mathbf{B} = \mathbf{AR}$: $t, \mathbf{v} - \begin{bmatrix} \mathbf{r} \\ \mathbf{0} \end{bmatrix}$: $\mathbf{m}$ PP : $(\mathbf{A}, \mathbf{D}, \mathbf{u}, \mathbf{G} = \mathbf{I} \otimes [b^0 | \ldots | b^{k-1}])$

$$\begin{bmatrix} \mathbf{A} & t\mathbf{G} - \mathbf{B} \end{bmatrix} \mathbf{v} = \mathbf{u} + \mathbf{A}\mathbf{r} + \begin{bmatrix} \mathbf{D} & \cdots \end{bmatrix} \mathbf{m}$$

- ✗ No random oracle. Needs different arguments for security proof
- ✓ Algebraic verification, handles arbitrary messages, security on standard assumptions

More Practical but Not Yet Practical Enough...

	Model	Assumptions	$\|\mathsf{sig}\|$	$\|\pi\|$
[LLM+16]	Adaptive	SIS/LWE	8617 KB	671581 KB
Ours [JRS23]	Adaptive	M-SIS/M-LWE	289 KB	660 KB

? How to optimize?

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- Relax security model [LLLW23][2]: **Selective security** (adversary tells what/how they will attack)

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- Relax security assumptions [BLNS23][3]: **Stronger assumptions** (optionally interactive)

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[BCR+23]	Adaptive	M-SIS/M-LWE	-	1878 KB

- Relax security model [LLLW23][2]: **Selective security** (adversary tells what/how they will attack)
- Relax security assumptions [BLNS23][3]: **Stronger assumptions** (optionally interactive)
- Optimize for implementation [BCR+23][4]: **Larger sizes**

? How to optimize **sizes and timings** while **keeping strong well-studied security?**

[2] Lai, Liu, Lysyanskaya, Wang. Lattice-based Commit-Transferrable Signatures and Applications to Anonymous Credentials. ePrint 2023/766

[3] Bootle, Lyubashevsky, Nguyen, Sorniotti. A Framework for Practical Anonymous Credentials from Lattices. Crypto 2023

[4] Blazy, Chevalier, Renaut, Ricosset, Sageloli, Senet. Efficient Implementation of a Post-Quantum Anonymous Credential Protocol. ARES 2023

❶ Change $\mathbf{B} = \mathbf{A}\mathbf{R}$ into $\mathbf{B} = \mathbf{A}\mathbf{R} + t^\star \mathbf{G}$ with hidden guess $t^\star$ on tag returned by $\mathcal{A}$
❷ Solve **SIS** instance $\mathbf{A}$ using the forgery $(t^\star, \mathbf{v}^\star)$ on fresh message $\mathbf{m}^\star$.

Step ❷ $$[\mathbf{A} | t^\star \mathbf{G} - \mathbf{B}]\mathbf{v}^\star = \mathbf{u} + \mathbf{D}\mathbf{m}^\star \iff \mathbf{A}((\mathbf{v}_1^\star - \mathbf{v}_1^{\mathcal{C}}) + \mathbf{R}(\mathbf{v}_2^\star - \mathbf{v}_2^{\mathcal{C}}) - \mathbf{S}(\mathbf{m}^\star - \mathbf{m})) = \mathbf{0}$$

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Step ❶

Sequence to change B

$\mathbf{AR} \longrightarrow \mathbf{U} \longrightarrow \mathbf{U} + t^\star\mathbf{G} \longrightarrow \mathbf{AR} + t^\star\mathbf{G}$

$\mathbf{AR}$	$\mathbf{U}$ → $\mathbf{U} + t^\star\mathbf{G}$	$\mathbf{AR} + t^\star\mathbf{G}$
Trapdoor ✓	**No trapdoor or ROM** (cannot answer queries) ✗	**Trapdoor** (except for $t^\star$) ✓

Statistical

"Unplayable" game but $\mathbf{AR}$ is statistically close to $\mathbf{AR} + t^\star\mathbf{G}$

Computational

$\mathbf{U}$ is an LWE challenge. Unplayable game... but we have to play it. Not poly-time

Use two trapdoors. $\mathbf{R}'$ used when $\mathbf{B}$ is uniform

$$\overline{\mathbf{A}}_t = \left[\mathbf{A} | t\mathbf{G} - \mathbf{B} | \mathbf{G} - \mathbf{A}\mathbf{R}'\right]$$

Second trapdoor slot
Dim: $d \times kd$
$(k = \log_b q)$

Partial Trapdoor Switching

Use two trapdoors. $\mathbf{R}'$ used when $\mathbf{B}$ is uniform

$$\overline{\mathbf{A}}_t = \left[\mathbf{A} | t\mathbf{G} - \mathbf{B} | \mathbf{G} - \mathbf{A}\mathbf{R}'\right]$$

Second trapdoor slot
Dim: $d \times kd$
$(k = \log_b q)$

We can do better by changing $\mathbf{B}$ **progressively**. First, split

$$\mathbf{G} = \mathbf{I}_d \otimes [b^0 | \dots | b^{k-1}] = [\mathbf{G}_1 \mid \dots \mid \mathbf{G}_d] \quad \text{with } \mathbf{G}_i = \mathbf{e}_i \otimes [b^0 | \dots | b^{k-1}]$$

$$\mathbf{R} = [\mathbf{R}_1 \mid \dots \mid \mathbf{R}_d] \quad \text{where } \mathbf{R}_i \text{ has } k \text{ columns}$$

$$t\mathbf{G} - \mathbf{B} = \left[t\mathbf{G}_1 - \mathbf{A}\mathbf{R}_1 \mid \dots \mid t\mathbf{G}_i - \mathbf{A}\mathbf{R}_i \mid \dots \mid t\mathbf{G}_d - \mathbf{A}\mathbf{R}_d\right]$$

$$t\mathbf{G}_i - \mathbf{A}\mathbf{R}_i \;\downarrow\; t\mathbf{G}_i - \mathbf{U}_i \longrightarrow \text{handled with } \mathbf{G}_i - \mathbf{A}\mathbf{R}'_i$$

$$t\mathbf{G}_i - \mathbf{U}_i \;\downarrow\; t\mathbf{G}_i - (\mathbf{A}\mathbf{R}_i + t^\star \mathbf{G}_i)$$

We can do better by changing $\mathbf{B}$ progressively

$G_{1,0}$

Public Key: $\mathbf{B} = [\mathbf{AR}_1 \mid \mathbf{AR}_2 \mid \ldots \mid \mathbf{AR}_d]$

Extra Slot: $\mathbf{A}_3 \sim \text{Uniform}$

Effective Trapdoor: $\mathbf{R} = [\mathbf{R}_1 \mid \mathbf{R}_2 \mid \ldots \mid \mathbf{R}_d]$

Effective Tag: $\mathbf{T} = \text{diag}(t, t, \ldots, t)$

Initial Game

We can do better by changing $\mathbf{B}$ progressively

$G_{1,0}$

$\downarrow \quad \mathbf{A}_3 \to \mathbf{G}_1 - \mathbf{A}'_3$

$G_{1,1}$

Public Key: $\mathbf{B} = [\mathbf{A}\mathbf{R}_1 \mid \mathbf{A}\mathbf{R}_2 \mid \ldots \mid \mathbf{A}\mathbf{R}_d]$

Extra Slot: $\mathbf{A}_3 = \mathbf{G}_1 - \mathbf{A}'_3$ $\quad (\mathbf{A}'_3 \sim \text{Unif.})$

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Hide partial gadget in $\mathbf{A}_3$: **Identical**

Partial Trapdoor Switching

We can do better by changing $\mathbf{B}$ progressively

$G_{1,0}$
$\downarrow \ \mathbf{A}_3 \to \mathbf{G}_1 - \mathbf{A}_3'$
$G_{1,1}$
$\downarrow \ \mathbf{A}_3' \to \mathbf{A}\mathbf{R}_1'$
$G_{1,2}$

Public Key: $\mathbf{B} = [\mathbf{A}\mathbf{R}_1 \mid \mathbf{A}\mathbf{R}_2 \mid \ldots \mid \mathbf{A}\mathbf{R}_d]$

Extra Slot: $\mathbf{A}_3 = \mathbf{G}_1 - \mathbf{A}\mathbf{R}_1'$

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Effective Tag: $\mathbf{T} = \mathrm{diag}(t, t, \ldots, t)$

Hide short relation in $\mathbf{A}_3$: **LWE**

Partial Trapdoor Switching

We can do better by changing $\mathbf{B}$ progressively

$G_{1,0}$
$\downarrow$ $\mathbf{A}_3 \to \mathbf{G}_1 - \mathbf{A}_3'$
$G_{1,1}$
$\downarrow$ $\mathbf{A}_3' \to \mathbf{A}\mathbf{R}_1'$
$G_{1,2}$
$\downarrow$ signatures use $\mathbf{R}_1'$
$G_{1,3}$

Public Key: $\mathbf{B} = [\mathbf{A}\mathbf{R}_1 \mid \mathbf{A}\mathbf{R}_2 \mid \ldots \mid \mathbf{A}\mathbf{R}_d]$

Extra Slot: $\mathbf{A}_3 = \mathbf{G}_1 - \mathbf{A}\mathbf{R}_1'$

Effective Trapdoor: $\mathbf{R} = [\mathbf{R}_1' \mid \mathbf{R}_2 \mid \ldots \mid \mathbf{R}_d]$

Effective Tag: $\mathbf{T} = \mathrm{diag}(1, t, \ldots, t)$

Sample signatures with $\mathbf{R}_1'$ instead of $\mathbf{R}_1$: **Trapdoor switching lemma**

We can do better by changing $\mathbf{B}$ progressively

$G_{1,0}$
$\downarrow$ $\mathbf{A}_3 \to \mathbf{G}_1 - \mathbf{A}'_3$
$G_{1,1}$
$\downarrow$ $\mathbf{A}'_3 \to \mathbf{A}\mathbf{R}'_1$
$G_{1,2}$
$\downarrow$ signatures use $\mathbf{R}'_1$
$G_{1,3}$
$\downarrow$ $\mathbf{A}\mathbf{R}_1 \to \mathbf{U}_1$
$G_{1,4}$

Public Key: $\mathbf{B} = [\mathbf{U}_1 \mid \mathbf{A}\mathbf{R}_2 \mid \ldots \mid \mathbf{A}\mathbf{R}_d]$ $(\mathbf{U}_1 \sim \mathrm{Unif.})$

Extra Slot: $\mathbf{A}_3 = \mathbf{G}_1 - \mathbf{A}\mathbf{R}'_1$

Effective Trapdoor: $\mathbf{R} = [\mathbf{R}'_1 \mid \mathbf{R}_2 \mid \ldots \mid \mathbf{R}_d]$

Effective Tag: $\mathbf{T} = \mathrm{diag}(1, t, \ldots, t)$

Remove short relation from $\mathbf{B}_1$: **LWE**

We can do better by changing $\mathbf{B}$ progressively

Public Key:	$\mathbf{B} = [\mathbf{U}_1' + t^\star \mathbf{G}_1 \mid \mathbf{AR}_2 \mid \ldots \mid \mathbf{AR}_d]$	$(\mathbf{U}_1' \sim \text{Unif.})$
Extra Slot:	$\mathbf{A}_3 = \mathbf{G}_1 - \mathbf{AR}_1'$	
Effective Trapdoor:	$\mathbf{R} = [\mathbf{R}_1' \mid \mathbf{R}_2 \mid \ldots \mid \mathbf{R}_d]$	
Effective Tag:	$\mathbf{T} = \text{diag}(1, t, \ldots, t)$	

$G_{1,0}$
$\downarrow$ $\mathbf{A}_3 \to \mathbf{G}_1 - \mathbf{A}_3'$
$G_{1,1}$
$\downarrow$ $\mathbf{A}_3' \to \mathbf{AR}_1'$
$G_{1,2}$
$\downarrow$ signatures use $\mathbf{R}_1'$
$G_{1,3}$
$\downarrow$ $\mathbf{AR}_1 \to \mathbf{U}_1$
$G_{1,4}$
$\downarrow$ $\mathbf{U}_1 \to \mathbf{U}_1' + t^\star \mathbf{G}_1$
$G_{1,5}$

Hide tag $t^\star$ with partial gadget in $\mathbf{B}_1$: **Identical**

We can do better by changing $\mathbf{B}$ progressively

Public Key:	$\mathbf{B} = [\mathbf{A}\mathbf{R}_1 + t^\star \mathbf{G}_1 \mid \mathbf{A}\mathbf{R}_2 \mid \ldots \mid \mathbf{A}\mathbf{R}_d]$
Extra Slot:	$\mathbf{A}_3 = \mathbf{G}_1 - \mathbf{A}\mathbf{R}_1'$
Effective Trapdoor:	$\mathbf{R} = [\mathbf{R}_1' \mid \mathbf{R}_2 \mid \ldots \mid \mathbf{R}_d]$
Effective Tag:	$\mathbf{T} = \mathrm{diag}(1, t, \ldots, t)$

$$G_{1,0} \xrightarrow{\mathbf{A}_3 \to \mathbf{G}_1 - \mathbf{A}_3'} G_{1,1} \xrightarrow{\mathbf{A}_3' \to \mathbf{A}\mathbf{R}_1'} G_{1,2} \xrightarrow{\text{signatures use } \mathbf{R}_1'} G_{1,3} \xrightarrow{\mathbf{A}\mathbf{R}_1 \to \mathbf{U}_1} G_{1,4} \xrightarrow{\mathbf{U}_1 \to \mathbf{U}_1' + t^\star \mathbf{G}_1} G_{1,5} \xrightarrow{\mathbf{U}_1' \to \mathbf{A}\mathbf{R}_1} G_{1,6}$$

Hide short relation in $\mathbf{B}_1$: **LWE**

We can do better by changing **B** progressively

$G_{1,0}$
$\downarrow$ $\mathbf{A}_3 \to \mathbf{G}_1 - \mathbf{A}'_3$
$G_{1,1}$
$\downarrow$ $\mathbf{A}'_3 \to \mathbf{A}\mathbf{R}'_1$
$G_{1,2}$
$\downarrow$ signatures use $\mathbf{R}'_1$
$G_{1,3}$
$\downarrow$ $\mathbf{A}\mathbf{R}_1 \to \mathbf{U}_1$
$G_{1,4}$
$\downarrow$ $\mathbf{U}_1 \to \mathbf{U}'_1 + t^\star \mathbf{G}_1$
$G_{1,5}$
$\downarrow$ $\mathbf{U}'_1 \to \mathbf{A}\mathbf{R}_1$
$G_{1,6}$
$\downarrow$ signatures use $\mathbf{R}_1$
$G_{1,7}$

Public Key: $\mathbf{B} = [\mathbf{A}\mathbf{R}_1 + t^\star \mathbf{G}_1 \mid \mathbf{A}\mathbf{R}_2 \mid \ldots \mid \mathbf{A}\mathbf{R}_d]$

Extra Slot: $\mathbf{A}_3 = \mathbf{G}_1 - \mathbf{A}\mathbf{R}'_1$

Effective Trapdoor: $\mathbf{R} = [\mathbf{R}_1 \mid \mathbf{R}_2 \mid \ldots \mid \mathbf{R}_d]$

Effective Tag: $\mathbf{T} = \mathrm{diag}(t - t^\star, t, \ldots, t)$

Sample signatures with $\mathbf{R}_1$ instead of $\mathbf{R}'_1$: **Trapdoor switching lemma**

Partial Trapdoor Switching

We can do better by changing $\mathbf{B}$ progressively

Public Key: $\mathbf{B} = [\mathbf{A}\mathbf{R}_1 + t^\star \mathbf{G}_1 \mid \mathbf{A}\mathbf{R}_2 \mid \ldots \mid \mathbf{A}\mathbf{R}_d]$

Extra Slot: $\mathbf{A}_3 = \mathbf{G}_1 - \mathbf{A}_3'$ $(\mathbf{A}_3' \sim \mathrm{Unif.})$

Effective Trapdoor: $\mathbf{R} = [\mathbf{R}_1 \mid \mathbf{R}_2 \mid \ldots \mid \mathbf{R}_d]$

Effective Tag: $\mathbf{T} = \mathrm{diag}(t - t^\star, t, \ldots, t)$

$G_{1,0}$
$\downarrow$ $\mathbf{A}_3 \to \mathbf{G}_1 - \mathbf{A}_3'$
$G_{1,1}$
$\downarrow$ $\mathbf{A}_3' \to \mathbf{A}\mathbf{R}_1'$
$G_{1,2}$
$\downarrow$ signatures use $\mathbf{R}_1'$
$G_{1,3}$
$\downarrow$ $\mathbf{A}\mathbf{R}_1 \to \mathbf{U}_1$
$G_{1,4}$
$\downarrow$ $\mathbf{U}_1 \to \mathbf{U}_1' + t^\star \mathbf{G}_1$
$G_{1,5}$
$\downarrow$ $\mathbf{U}_1' \to \mathbf{A}\mathbf{R}_1$
$G_{1,6}$
$\downarrow$ signatures use $\mathbf{R}_1$
$G_{1,7}$
$\downarrow$ $\mathbf{A}\mathbf{R}_1' \to \mathbf{A}_3'$
$G_{1,8}$

Remove short relation from $\mathbf{A}_3$: **LWE**

Partial Trapdoor Switching

We can do better by changing $\mathbf{B}$ progressively

Public Key: $\mathbf{B} = [\mathbf{A}\mathbf{R}_1 + t^\star \mathbf{G}_1 \mid \mathbf{A}\mathbf{R}_2 \mid \ldots \mid \mathbf{A}\mathbf{R}_d]$

Extra Slot: $\mathbf{A}_3 \sim \text{Uniform}$

Effective Trapdoor: $\mathbf{R} = [\mathbf{R}_1 \mid \mathbf{R}_2 \mid \ldots \mid \mathbf{R}_d]$

Effective Tag: $\mathbf{T} = \text{diag}(t - t^\star, t, \ldots, t)$

$G_{1,0}$
$\downarrow$ $\mathbf{A}_3 \to \mathbf{G}_1 - \mathbf{A}_3'$
$G_{1,1}$
$\downarrow$ $\mathbf{A}_3' \to \mathbf{A}\mathbf{R}_1'$
$G_{1,2}$
$\downarrow$ signatures use $\mathbf{R}_1'$
$G_{1,3}$
$\downarrow$ $\mathbf{A}\mathbf{R}_1 \to \mathbf{U}_1$
$G_{1,4}$
$\downarrow$ $\mathbf{U}_1 \to \mathbf{U}_1' + t^\star \mathbf{G}_1$
$G_{1,5}$
$\downarrow$ $\mathbf{U}_1' \to \mathbf{A}\mathbf{R}_1$
$G_{1,6}$
$\downarrow$ signatures use $\mathbf{R}_1$
$G_{1,7}$
$\downarrow$ $\mathbf{A}\mathbf{R}_1' \to \mathbf{A}_3'$
$G_{1,8}$
$\downarrow$ $\mathbf{G}_1 - \mathbf{A}_3' \to \mathbf{A}_3$
$G_{1,9}$

Remove partial gadget from $\mathbf{A}_3$: **Identical**

We then loop the hybrid argument until we changed every slot

$$
\begin{array}{rc}
 & G_{1,0} \\
\mathbf{A}_3 \to \mathbf{G}_1 - \mathbf{A}'_3 & \downarrow \\
 & G_{1,1} \\
\mathbf{A}'_3 \to \mathbf{A}\mathbf{R}'_1 & \downarrow \\
 & G_{1,2} \\
\text{signatures use } \mathbf{R}'_1 & \downarrow \\
 & G_{1,3} \\
\mathbf{A}\mathbf{R}_1 \to \mathbf{U}_1 & \downarrow \\
 & G_{1,4} \\
\mathbf{U}_1 \to \mathbf{U}'_1 + t^\star \mathbf{G}_1 & \downarrow \\
 & G_{1,5} \\
\mathbf{U}'_1 \to \mathbf{A}\mathbf{R}_1 & \downarrow \\
 & G_{1,6} \\
\text{signatures use } \mathbf{R}_1 & \downarrow \\
 & G_{1,7} \\
\mathbf{A}\mathbf{R}'_1 \to \mathbf{A}'_3 & \downarrow \\
 & G_{1,8} \\
\mathbf{G}_1 - \mathbf{A}'_3 \to \mathbf{A}_3 & \downarrow \\
 & G_{1,9}
\end{array}
$$

$G_{1,9} = G_{2,0}$

$$
\begin{array}{rc}
 & G_{2,0} \\
\mathbf{A}_3 \to \mathbf{G}_2 - \mathbf{A}'_3 & \downarrow \\
 & G_{2,1} \\
\mathbf{A}'_3 \to \mathbf{A}\mathbf{R}'_2 & \downarrow \\
 & G_{2,2} \\
\text{signatures use } \mathbf{R}'_2 & \downarrow \\
 & G_{2,3} \\
\mathbf{A}\mathbf{R}_2 \to \mathbf{U}_2 & \downarrow \\
 & G_{2,4} \\
\mathbf{U}_2 \to \mathbf{U}'_2 + t^\star \mathbf{G}_2 & \downarrow \\
 & G_{2,5} \\
\mathbf{U}'_2 \to \mathbf{A}\mathbf{R}_2 & \downarrow \\
 & G_{2,6} \\
\text{signatures use } \mathbf{R}_2 & \downarrow \\
 & G_{2,7} \\
\mathbf{A}\mathbf{R}'_2 \to \mathbf{A}'_3 & \downarrow \\
 & G_{2,8} \\
\mathbf{G}_2 - \mathbf{A}'_3 \to \mathbf{A}_3 & \downarrow \\
 & G_{2,9}
\end{array}
$$

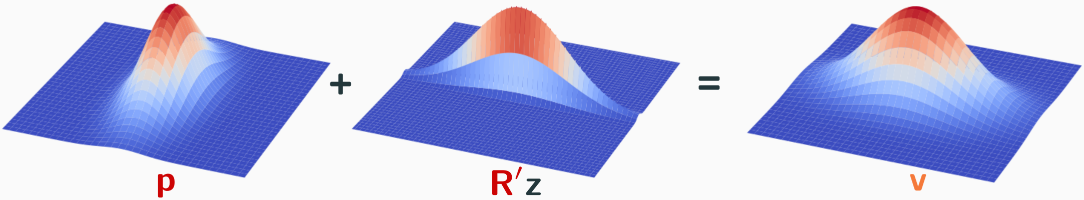
$\cdots$

$\cdots = G_{d,0}$

$$
\begin{array}{rc}
 & G_{d,0} \\
\mathbf{A}_3 \to \mathbf{G}_d - \mathbf{A}'_3 & \downarrow \\
 & G_{d,1} \\
\mathbf{A}'_3 \to \mathbf{A}\mathbf{R}'_d & \downarrow \\
 & G_{d,2} \\
\text{signatures use } \mathbf{R}'_d & \downarrow \\
 & G_{d,3} \\
\mathbf{A}\mathbf{R}_d \to \mathbf{U}_d & \downarrow \\
 & G_{d,4} \\
\mathbf{U}_d \to \mathbf{U}'_d + t^\star \mathbf{G}_d & \downarrow \\
 & G_{d,5} \\
\mathbf{U}'_d \to \mathbf{A}\mathbf{R}_d & \downarrow \\
 & G_{d,6} \\
\text{signatures use } \mathbf{R}_d & \downarrow \\
 & G_{d,7} \\
\mathbf{A}\mathbf{R}'_d \to \mathbf{A}'_3 & \downarrow \\
 & G_{d,8} \\
\mathbf{G}_d - \mathbf{A}'_3 \to \mathbf{A}_3 & \downarrow \\
 & G_{d,9}
\end{array}
$$

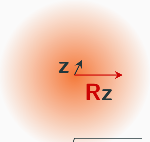
Use **elliptical Gaussians** instead of spherical

$$\begin{bmatrix} s_1^2\mathbf{I} & \\ & s_2^2\mathbf{I} \end{bmatrix} - s_z^2\mathbf{R}'\mathbf{R}'^T \qquad s_z^2\mathbf{R}'\mathbf{R}'^T \qquad \begin{bmatrix} s_1^2\mathbf{I} & \\ & s_2^2\mathbf{I} \end{bmatrix}$$

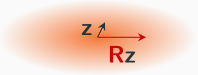


Spherical Sampling

Elliptical Sampling



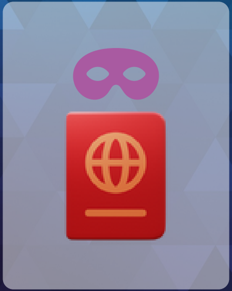
$$\mathbf{v} = \mathbf{p} + \begin{bmatrix} \mathbf{Rz} \\ \mathbf{z} \end{bmatrix}$$



$s \approx s_z\sqrt{1 + \|\mathbf{R}\|_2^2}$

$s_1 \approx s_z\|\mathbf{R}\|_2, \quad s_2 \approx s_z$

Anonymous Credentials Use-Case: Implementation & Performance



Estimated Performance

	Model	Assumptions	|sig|	$\|\pi\|$
[LLM+16]	Adaptive	SIS/LWE	8617 KB	671581 KB
Ours [JRS23]	Adaptive	M-SIS/M-LWE	289 KB	660 KB
[LLLW23]	Selective	M-SIS/M-LWE	118 KB	193 KB
[BLNS23]-1	Adaptive	NTRU-ISIS$_f$	72 KB	243 KB
[BLNS23]-2	Adaptive	Int-NTRU-ISIS$_f$	3.5 KB	62 KB
[BCR+23]	Adaptive	M-SIS/M-LWE	-	1878 KB
Ours [AGJ+24]	Adaptive	M-SIS/M-LWE	6.8 KB	79 KB

Further (quick) optimizations?

Estimated Performance

	Model	Assumptions	$|\mathrm{sig}|$	$|\pi|$
[LLM+16]	Adaptive	SIS/LWE	8617 KB	671581 KB
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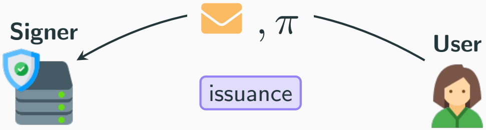
Further (quick) optimizations?

- Reducing garbage commitments [LNP22] $\longrightarrow$ 77 KB (3% gain)
- Dilithium compression for commitments [LNP22] $\longrightarrow$ 70 KB (9% gain)
- Bimodal rejection sampling [LN22][5] $\longrightarrow$ 61 KB (13% gain)

Estimations give $|\pi| \approx 61$ KB (overall 24% gain), while on **standard assumptions**

[5] Lyubashevsky, Nguyen. BLOOM: Bimodal Lattice One-Out-of-Many Proofs and Applications. Asiacrypt 2022



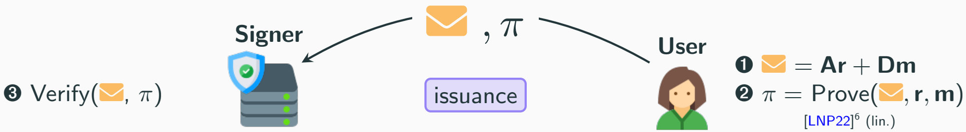
❶ $= \mathbf{Ar} + \mathbf{Dm}$

❷ $\pi = \mathsf{Prove}(\,, \mathbf{r}, \mathbf{m})$

[LNP22][6] (lin.)

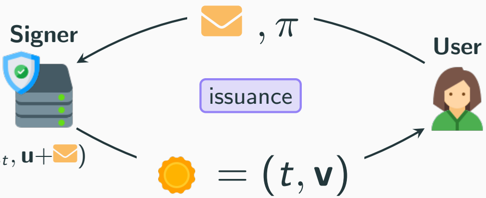
Step	❶	❷	❸	❹+❺	❻	Total
Avg. Time	1 ms	222 ms				

[6] Lyubashevsky, Nguyen, Plançon. Lattice-Based Zero-Knowledge Proofs and Applications: Shorter, Simpler, and More General. Crypto 2022



Step	❶	❷	❸	❹+❺	❻	Total
Avg. Time	1 ms	222 ms	101 ms			

[6] Lyubashevsky, Nguyen, Plançon. Lattice-Based Zero-Knowledge Proofs and Applications: Shorter, Simpler, and More General. Crypto 2022

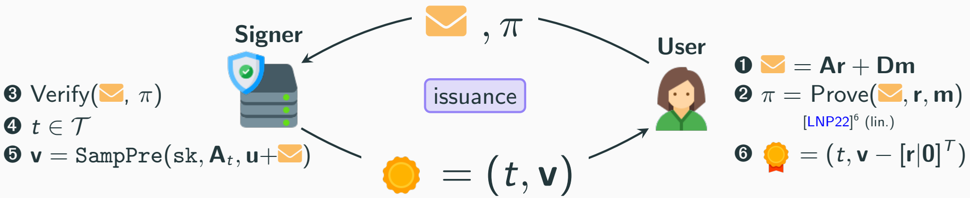


❸ Verify(, π)
❹ $t \in \mathcal{T}$
❺ $\mathbf{v} = \texttt{SampPre}(\mathtt{sk}, \mathbf{A}_t, \mathbf{u}+$)

❶ $= \mathbf{Ar} + \mathbf{Dm}$
❷ $\pi = \mathsf{Prove}($ $, \mathbf{r}, \mathbf{m})$
[LNP22][6] (lin.)

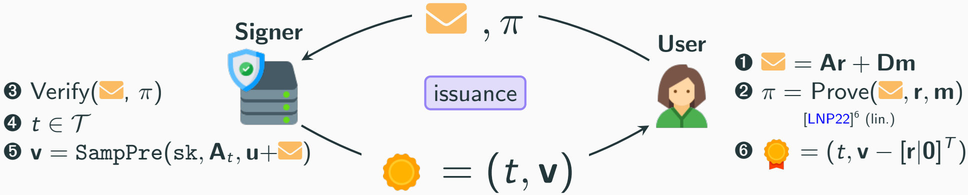
Step	❶	❷	❸	❹+❺	❻	Total
Avg. Time	1 ms	222 ms	101 ms	57 ms		

[6] Lyubashevsky, Nguyen, Plançon. Lattice-Based Zero-Knowledge Proofs and Applications: Shorter, Simpler, and More General. Crypto 2022



Step	❶	❷	❸	❹+❺	❻	Total
Avg. Time	1 ms	222 ms	101 ms	57 ms	2 ms	

[6] Lyubashevsky, Nguyen, Plançon. Lattice-Based Zero-Knowledge Proofs and Applications: Shorter, Simpler, and More General. Crypto 2022



Step	❶	❷	❸	❹+❺	❻	Total
Avg. Time	1 ms	222 ms	101 ms	57 ms	2 ms	**383 ms**

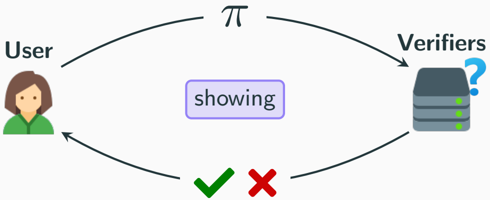
✔ Full issuance takes less than half a second! **Imperceptible on user experience**.

[6] Lyubashevsky, Nguyen, Plançon. Lattice-Based Zero-Knowledge Proofs and Applications: Shorter, Simpler, and More General. Crypto 2022

Step	❶	❷	Total
Avg. Time ([BCR+23])	1843 ms		
Avg. Time (Ours [AGJ+24])	357 ms		

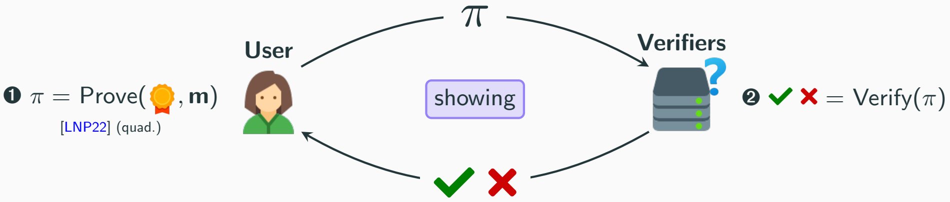
❶ $\pi = \mathsf{Prove}($🏅$, \mathbf{m})$

[LNP22] (quad.)



❷ ✔ ✘ $= \mathsf{Verify}(\pi)$

Step	❶	❷	Total
Avg. Time ([BCR+23])	1843 ms	172 ms	
Avg. Time (Ours [AGJ+24])	357 ms	147 ms	



Step	❶	❷	Total
Avg. Time ([BCR+23])	1843 ms	172 ms	2015 ms
Avg. Time (Ours [AGJ+24])	357 ms	147 ms	**504 ms**

✔ Full showing takes around half a second! $4\times$ faster than [BCR+23].

Conclusion and Directions

Wrapping Up

General-Purpose Framework for **Privacy-Enhanced Lattice Signature**

- Based on standard post-quantum assumptions (M-SIS, M-LWE)
- Relatively compact for Digital Identity use-cases
- Concretely efficient with a proof-of-concept implementation

Perspectives

- Optimizations in specific constructions? (ePrint 2024/1289 for blind signatures)
- Use of approximate trapdoors for compactness? (ePrint 2024/1952, talk on Mar. 19)
- Is the partial trapdoor slot necessary?
- MPC-in-the-Head to construct more efficient lattice ZKP?

- Implement optimizations of ZKP (garbage, compression, bimodal): Done for BS
- Optimized implementation (dedicated backend, parallelization, parameter selection)

Wrapping Up

General-Purpose Framework for **Privacy-Enhanced Lattice Signature**

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Thank You!

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