Phoenix: Hash-and-Sign with Aborts from Lattice Gadgets

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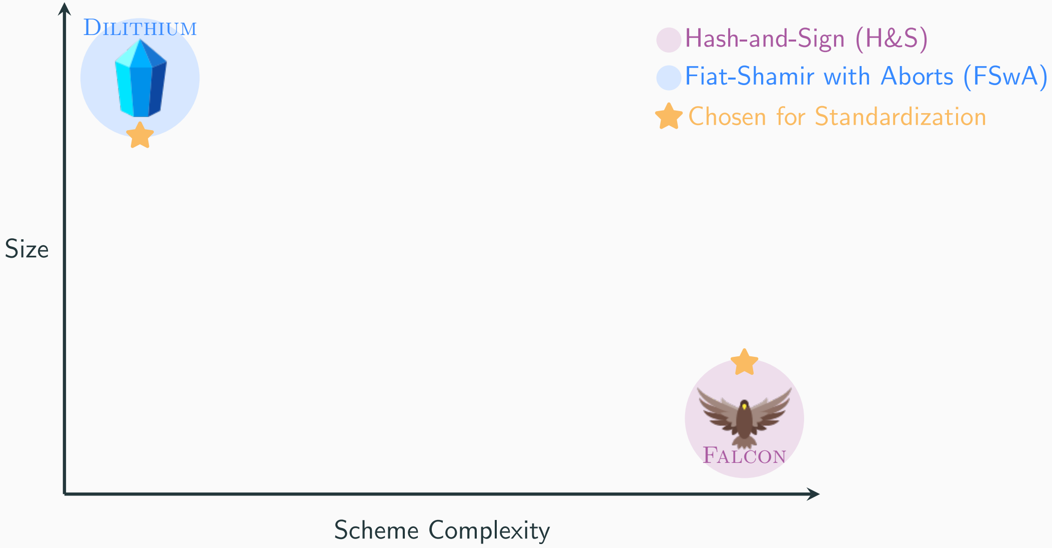
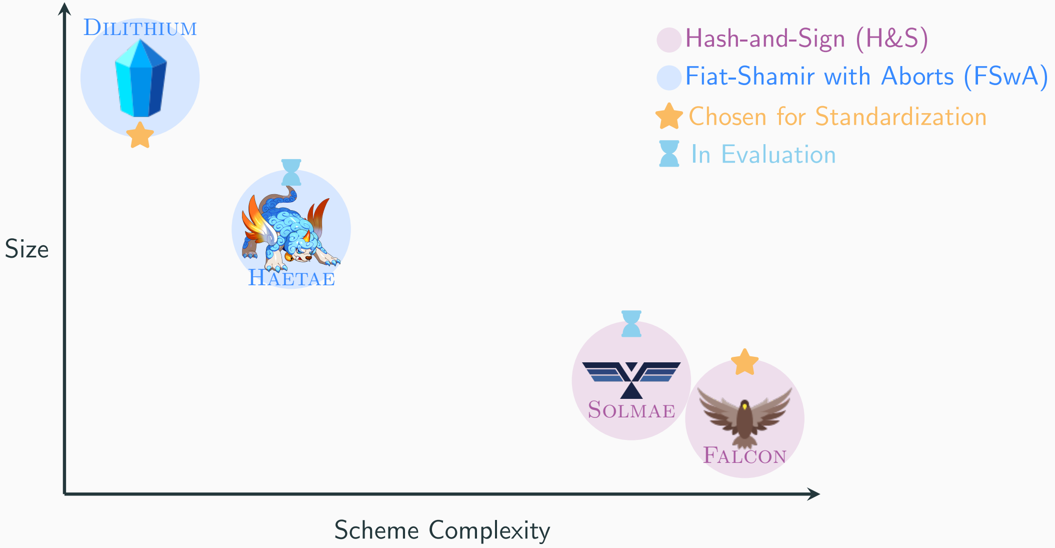
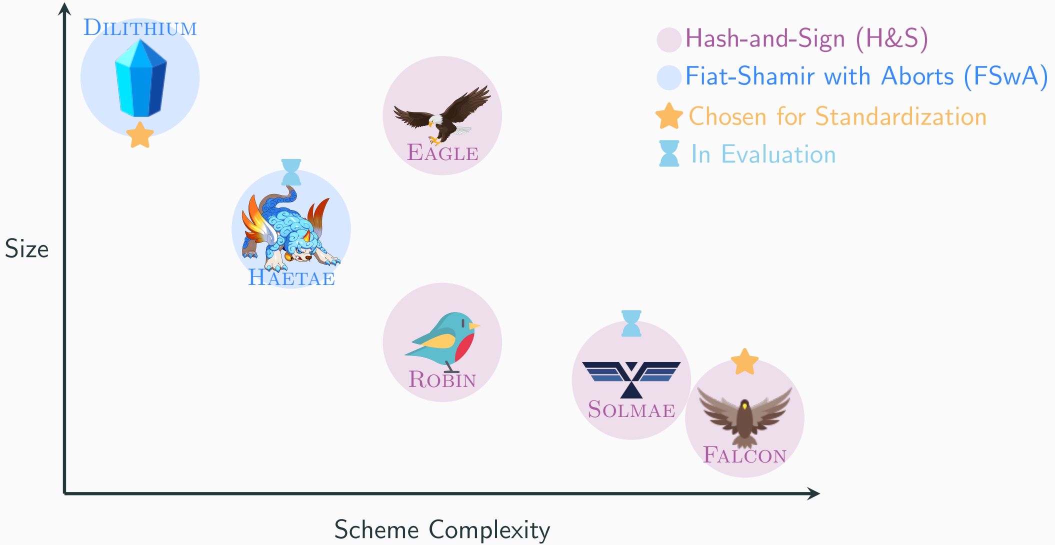
PQCrypto 2024 - June 13th, 2024

Lattice-Based Signatures




Dilithium
Haetae
Solmae
Falcon
Size
Scheme Complexity
Hash-and-Sign (H&S)
Fiat-Shamir with Aborts (FSwA)
Chosen for Standardization
In Evaluation

Lattice-Based Signatures



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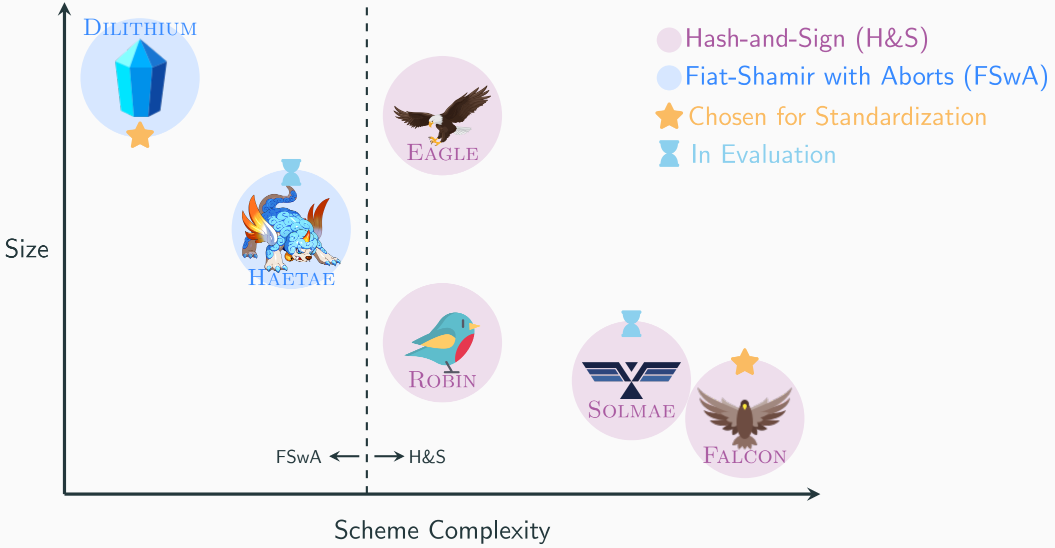
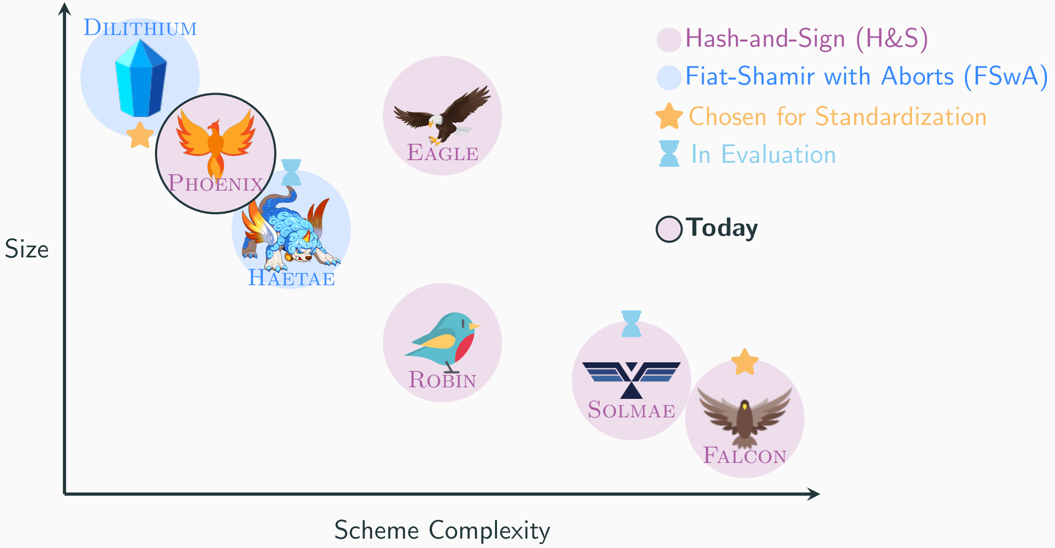


Lattice-Based Signatures



$\mathrm{ISIS}_{m,d,q,\beta}$

Given $(\mathbf{A}, \mathbf{u}) \hookleftarrow U(R_q^{d\times m+1})$, find $\mathbf{x} \in R^m$ such that $\mathbf{A}\mathbf{x} = \mathbf{u} \bmod q$, $\|\mathbf{x}\|_2 \leq \beta$.

ISIS is hard unless we know a trapdoor **R** on **A**.

- Ability to invert $f_{\mathbf{A}} : \mathbf{x} \mapsto \mathbf{A}\mathbf{x} \bmod q$ over bounded domain
  - Ability to randomize preimage finding without leaking **R** → **Preimage Sampling**
    - Design secure hash-and-sign signatures [GPV08][1]

Several choices for trapdoors and preimage samplers. **Today:** Gadget-based Trapdoors

[1] Gentry, Peikert, Vaikuntanathan. Trapdoors for Hard Lattices and New Cryptographic Constructions. STOC 2008.

Trapdoor Preimage Sampling with Aborts

Approaches to Gadget-Based Samplers

Micciancio-Peikert trapdoors [MP12][2]: Family of matrices $\overline{\mathbf{A}}$ such that

$$\overline{\mathbf{A}}\mathbf{R}' = \mathbf{G} \bmod q, \quad \text{with} \quad \mathbf{R}' = \begin{bmatrix} \mathbf{R} \\ \mathbf{I} \end{bmatrix}, \quad \text{i.e. } \overline{\mathbf{A}} = [\mathbf{A}|\mathbf{G} - \mathbf{A}\mathbf{R}] \text{ and } \mathbf{A} = [\mathbf{I}|\mathbf{A}']$$

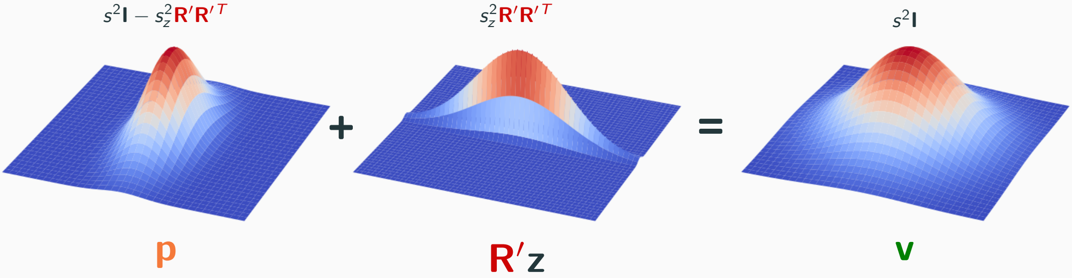
with $\mathbf{G}$ a public gagdet matrix allowing for *efficient short inversion*. $\mathbf{B} = \mathbf{A}\mathbf{R}$.

Naive Approach: Compute $\mathbf{z}$ so that $\mathbf{G}\mathbf{z} = \mathbf{u} \bmod q$, and return $\mathbf{R}'\mathbf{z}$ as preimage of $\mathbf{u}$.
Typically, $\boldsymbol{G} = \boldsymbol{I} \otimes [b^0|\ldots|b^{k-1}]$ with $k = \log_b q$.

- ✗ Collecting many preimages will leak $\mathbf{R}$...
- Add a mask $\mathbf{p}$ to get preimages $\mathbf{v} = \mathbf{p} + \mathbf{R}'\mathbf{z}$ (and gadget inversion on $\mathbf{u} - \overline{\mathbf{A}}\mathbf{p}$ instead of $\mathbf{u}$)

[2] Micciancio, Peikert. Trapdoors for Lattices: Simpler, Tighter, Faster, Smaller. Eurocrypt 2012.

Compensate the statistical leakage by adapting the covariance of **p** [MP12]. Only available analysis for **z** and **p** Gaussian.



Convolution: compact, but Gaussian gadget sampling for **z** and complex non-spherical Gaussian sampling for **p**.

How to Choose the Mask? (2) Rejection

What do we need to hide exactly?

$$\mathbf{p} + \mathbf{R}'\mathbf{z} = \begin{bmatrix} \mathbf{p}_1 + \mathbf{R}\mathbf{z} \\ \mathbf{p}_2 + \mathbf{z} \end{bmatrix}$$

- $\mathbf{p}_1 + \mathbf{R}\mathbf{z}$ → Shift to hide
- $\mathbf{p}_2 + \mathbf{z}$ → Leaks information on shift

Cannot set $\mathbf{p}_2 = \mathbf{0}$...

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⚠ **Cannot set $\mathbf{p}_2 = \mathbf{0}$...** What if we really want $\mathbf{p}_2 = \mathbf{0}$?

	CMT	CHAL	RESP
Fiat-Shamir	$\mathbf{A}\mathbf{y}$	$\mathbf{c}$	$\mathbf{y} + \mathbf{S}\mathbf{c}$
$\mathbf{p}_2 = \mathbf{0}$	$\mathbf{A}\mathbf{p}_1$	$\mathbf{z}$	$\mathbf{p}_1 + \mathbf{R}\mathbf{z}$

⚠ **Leaking Secret**

? What do we need to hide exactly?

$$\mathbf{p} + \mathbf{R}'\mathbf{z} = \begin{bmatrix} \mathbf{p}_1 + \mathbf{R}\mathbf{z} \\ \mathbf{p}_2 + \mathbf{z} \end{bmatrix}$$

- $\mathbf{R}\mathbf{z}$ → Shift to hide
- $\mathbf{z}$ → Leaks information on shift

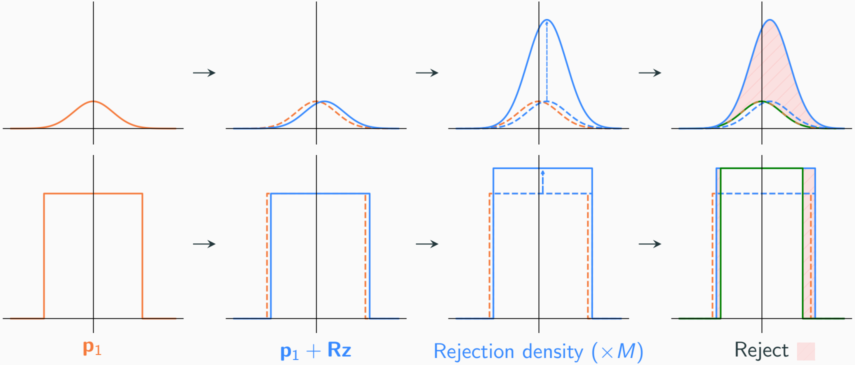
⚠ **Cannot set $\mathbf{p}_2 = \mathbf{0}$...** What if we really want $\mathbf{p}_2 = \mathbf{0}$?

	CMT	CHAL	RESP
Fiat-Shamir with aborts	$\mathbf{A}\mathbf{y}$	$\mathbf{c}$	$\mathbf{y} + \mathbf{S}\mathbf{c}$ & Rejection
$\mathbf{p}_2 = \mathbf{0}$	$\mathbf{A}\mathbf{p}_1$	$\mathbf{z}$	$\mathbf{p}_1 + \mathbf{R}\mathbf{z}$ & Rejection

✔ **Rejection Sampling**

How to Choose the Mask? (2) Rejection

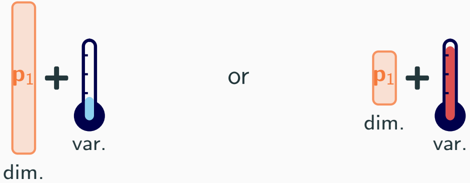
Set $\mathbf{p}_2 = \mathbf{0}$, $\mathbf{z} = \mathbf{G}^{-1}(\mathbf{u} - \mathbf{A}\mathbf{p}_1)$, and reject $\mathbf{p}_1$ if there is statistical leakage [LW15][3].



Rejection: versatile, but needs statistical regularity of $\mathbf{u} - \mathbf{A}\mathbf{p}_1$ (or $\mathbf{A}\mathbf{p}_1$ if $\mathbf{u}$ arbitrary).

[3] Lyubashevsky, Wichs. Simple lattice trapdoor sampling from a broad class of distributions. PKC 2015.

Statistical regularity needs high entropy $\mathbf{p}_1$



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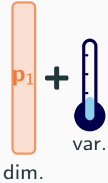
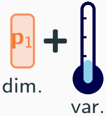


or

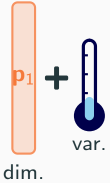
Leverage entropy of the **non-arbitrary syndrome** to avoid regularity argument of [LW15]

With $\mathbf{u} = \mathcal{H}(m)$, no need for high entropy $\mathbf{p}_1$



Rejection Sampler for Uniform Syndromes

Statistical regularity needs high entropy $\mathbf{p}_1$



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Leverage entropy of the **non-arbitrary syndrome** to avoid regularity argument of [LW15]

With $\mathbf{u} = \mathcal{H}(m)$, no need for high entropy $\mathbf{p}_1$

- $\mathbf{p}_1 \hookleftarrow \mathscr{P}_s$ (source distribution)
- $\mathbf{v}_2 \leftarrow \mathbf{G}^{-1}(\mathbf{u} - \mathbf{A}\mathbf{p}_1)$ and $\mathbf{v}_1 \leftarrow \mathbf{p}_1 + \mathbf{R}\mathbf{v}_2$
- $\mathrm{Rej}(\mathbf{p}_1, \mathbf{v}_1, \mathscr{P}_s, \mathscr{P}_t)$
- Output $\mathbf{v} = (\mathbf{v}_1, \mathbf{v}_2)$

verifies $\overline{\mathbf{A}}\mathbf{v} = \mathbf{u}$

Rejection Sampler

Combination with approximate trapdoors [CGM19][4]: Finding $\mathbf{v}'$ s.t. $\overline{\mathbf{A}}\mathbf{v}' + \mathbf{e} = \mathbf{u}$ with $\mathbf{e}$ small is sufficient. Let $\mathbf{G}_H = \mathbf{I} \otimes [b^{\ell} | \ldots | b^{k-1}]$ (high-order decomposition).

[4] Chen, Genise, Mukherjee. Approximate trapdoors for lattices and smaller hash-and-sign signatures. Asiacrypt 2019.

[5] Yu, Jia, Wang. Compact lattice gadget and its applications to hash-and-sign signatures. Crypto 2023.

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Approx. Rejection Sampler

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Approx. Rejection Sampler

Preimage error $\mathbf{e}$ bounded $b^\ell - 1$ and uniform

- ✓ Smaller than [CGM19]
- ✓ Allows for dropping more entries (up to $\mathbf{G}_H$ square with $\ell = k - 1$).
- ✗ Slightly larger than with semi-random sampler [YJW23][5], but much smaller $\mathbf{v}_2$.

[4] Chen, Genise, Mukherjee. Approximate trapdoors for lattices and smaller hash-and-sign signatures. Asiacrypt 2019.

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Application to Hash-and-Sign Signatures: Phoenix

Landscape of Hash-and-Sign Signatures

Lattice-based hash-and-sign signatures follow the GPV framework [GPV08]. Requires two main ingredients: a trapdoor (hidden under some assumption), a simulatable preimage sampler.

Trapdoor	Sampler	Scheme	Performance	Complexity	Assumption	Distribution
NTRU Trapdoors	Klein Sampler	Falcon	✓✓	✗✗	**NTRU**	Λ
	Hybrid Sampler	Mitaka, Solmae				
Gadget Trapdoors	Convolution Sampler	MP12, CGM19, GL20	✗	~	**LWE**, **iNTRU**	Λ
	Rejection Sampler	LW15	✗✗	✓	**LWE**	Λ ⊓ ···
	Semi-random Sampler	Robin	✓	~	**iNTRU**	Λ
		Eagle	~		**LWE**	

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Gadget Trapdoors	Convolution Sampler	MP12, CGM19, GL20	✗	~	**LWE**, **iNTRU**	
	Rejection Sampler	LW15	✗✗	✓	**LWE**	
		Phoenix	~			
	Semi-random Sampler	Robin	✓	~	**iNTRU**	
		Eagle	~		**LWE**	

MP Trapdoors

+

Approximate Rejection Sampler

+

GPV Framework

→ Phoe... **Flamingo**

? Short signature but large public key. Can we reduce the public key size?

Phoenix: Approximate Rejection Sampler and Key Compression

MP Trapdoors

+

Approximate Rejection Sampler

+

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Short signature but large public key. Can we reduce the public key size? **Yes!**

Split $= \mathbf{B}$ into $\mathbf{B}_L + 2^{\ell'}\mathbf{B}_H$.

$$\mathbf{v}_{1,1} + \mathbf{A}'\mathbf{v}_{1,2} + (\mathbf{G}_H - \mathbf{B})\mathbf{v}_2 = \mathcal{H}(m)$$

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$\mathbf{B}_L\mathbf{v}_2$ short compression error

$$\mathbf{v}_{1,1} + \mathbf{A}'\mathbf{v}_{1,2} + (\mathbf{G}_H - 2^{\ell'}\mathbf{B}_H)\mathbf{v}_2 - \mathbf{B}_L\mathbf{v}_2 = \mathcal{H}(m)$$

Phoenix: Approximate Rejection Sampler and Key Compression

MP Trapdoors

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Split $= \mathbf{B}$ into $\mathbf{B}_L + 2^{\ell'}\mathbf{B}_H$.

$\mathbf{v}'_{1,1}$ includes sampling+compression errors

$$\mathbf{v}'_{1,1} + \mathbf{A}'\mathbf{v}_{1,2} + (\mathbf{G}_H - 2^{\ell'}\mathbf{B}_H)\mathbf{v}_2 = \mathcal{H}(m)$$

Compression "for free" made possible by unusually short $\mathbf{v}_2$.
No additional hints / rejection sampling compared to FSwA key compression.

Performance

	NIST-II		NIST-III		NIST-V	
	|pk|	|sig|	|pk|	|sig|	|pk|	|sig|
Dilithium	1312	2420	1952	3293	2592	4595
Eagle (HuFu)	-	- (2455)	1952	3052 (3540)	-	- (4520)
Phoenix	1184	2190	1490	2897	2219	4468

Conclusion

Wrapping Up

Our contributions (https://ia.cr/2023/446)

- Rehabilitating the [LW15] sampler when targets are uniform
- Optimization with approximate trapdoors
- Application to Hash-and-Sign (with Aborts) with key compression for free: **Phoenix**
  - General distributions without complex Gaussian samplers
  - Interpolates performance of FSwA and Hash-and-Sign
  - Tighter QROM security than FSwA

Future Work

- Compact distributions with easy-to-protect rejection step?
- Other applications of the approximate rejection sampler?
- Approximate rejection sampler with iNTRU?

Thank you for your attention!

Questions?

Y. Chen, N. Genise, and P. Mukherjee.
Approximate Trapdoors for Lattices and Smaller Hash-and-Sign Signatures.
In ASIACRYPT, 2019.

C. Gentry, C. Peikert, and V. Vaikuntanathan.
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