Lattice Signature with Efficient Protocols, Application to Anonymous Credentials

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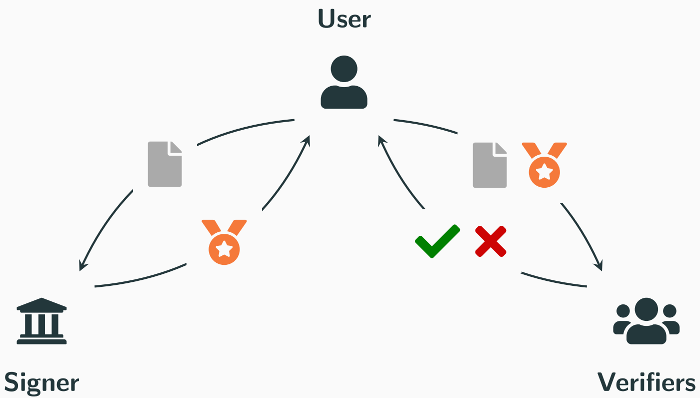
Signature with Efficient Protocols (SEP)



The message must be revealed to sign and verify. Not suited for privacy-enhancing applications.

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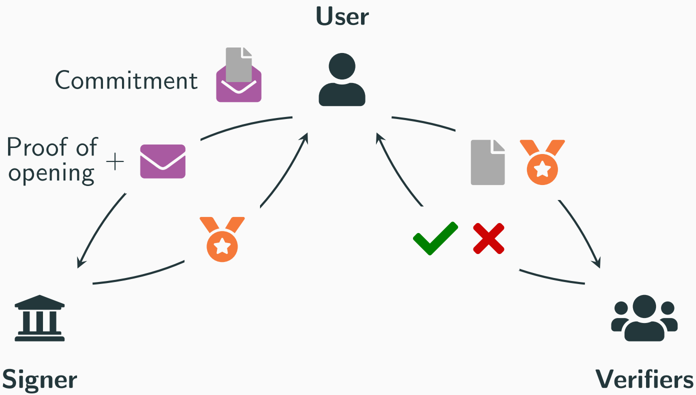

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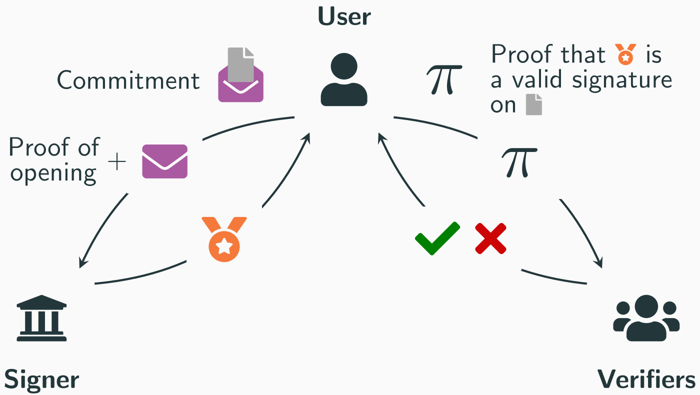

Neither nor have to be revealed, but need for Zero-Knowledge arguments, and "structure-preserving" signature.

An Interesting Versatility

Many concrete privacy-enhancing applications.

- **Anonymous Credentials Systems**: requires the ability to
  - ✓ sign committed messages
  - ✓ prove possession of a message-signature pair in ZK
- **Group Signatures**: requires to add a verifiable encryption of the user identity
- **Blind Signatures**: requires the ability to
  - ✓ sign committed messages
  - ✓ prove possession of a signature on a public message in ZK
- **E-Cash Systems**
- etc.

Real industrial impact: EPID and DAA deployed in billions of devices (TPM, SGX).
Blind/Group signatures in ISO standards

Existing Signatures with Efficient Protocols

Very efficient instantiations of SEPs in the classical setting.

- [CL02][1] Based on the Strong-RSA assumption.
- [CL04][2][BB08][3][PS16][4] Based on pairings in bilinear groups.

[BB08][PS16] are constant-size. Very efficient group signatures, anonymous credentials, etc.

- Best group signature is based on SEP: 0.16 KB

[1] J. Camenisch, A. Lysyanskaya. A signature scheme with efficient protocols. SCN 2002.
[2] J. Camenisch, A. Lysyanskaya. Signature schemes and anonymous credentials from bilinear maps. CRYPTO 2004.
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? Those are vulnerable to quantum computing. How about **post-quantum** solutions?

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Existing PQC Signature with Efficient Protocols

Only one proposal of post-quantum signature with efficient protocols:

- [LLM+16][5] Proof of concept based on standard lattices.

		$\lvert pk \rvert$	$\lvert sk \rvert$	$\lvert sig \rvert$	$\lvert \pi \rvert$
[LLM+16]	Exact Proof	3 TB	15 GB	9 MB	10 GB
	Appr. Proof	7 TB	37 GB	14 MB	670 MB

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Today

Simpler, more compact, more efficient construction on standard lattices, and extension to ideal and module lattices.

		$\|pk\|$	$\|sk\|$	$\|sig\|$	$\|\pi\|$
Ours	Exact Proof	**8 MB**	**9 MB**	**270 KB**	**640 KB**

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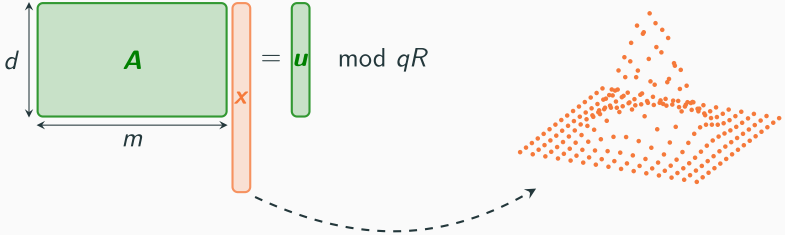
Warmup: Trapdoor and Regular Signatures

Module-SIS$_{m,d,q,\beta}$

Given $\boldsymbol{A} \hookleftarrow U((R/qR)^{d\times m})$, find a **non-zero** $\boldsymbol{x} \in R^m$ such that $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{0} \bmod qR$, $0 < \|\boldsymbol{x}\|_2 \leq \beta$.

$R = \mathbb{Z}[x]/\langle x^n + 1\rangle$ with $n = 2^k$

Trapdoor on $\boldsymbol{A}$: secret piece of information used to sample Gaussian vector $\boldsymbol{x}$ such that $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{u} \bmod qR$ for any syndrome $\boldsymbol{u}$



A Simple Signature Using Trapdoors

Design rationale of **Falcon** (future standard FN-DSA from NIST)

Alice has a public key $= \boldsymbol{A}$, and a **secret trapdoor** on $\boldsymbol{A}$.

Signing a message $\boldsymbol{m}$

❶ Hash the message into $\boldsymbol{u} = \mathcal{H}(\boldsymbol{m})$.

❷ Use to sample a Gaussian $\boldsymbol{x}$ () such that $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{u} \bmod qR$.

Signature is $= \boldsymbol{x}$.

Verifying a signature

❶ Hash the message into $\boldsymbol{u} = \mathcal{H}(\boldsymbol{m})$.

❷ Check that $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{u} \bmod qR$, and that $\boldsymbol{x}$ is short.

Our Lattice Signature With Efficient Protocols

1 **Original Construction** from [LLM+16]

🔑 $= \boldsymbol{T_A}$ (Trapdoor), 🔑 $= \boldsymbol{A}_i, \boldsymbol{u}, \boldsymbol{D}, \boldsymbol{D}_j$ uniform public

$\texttt{sig} = ((\tau_i)_i, \boldsymbol{v}, \boldsymbol{r})$ with τ_i tag bits, $\boldsymbol{v}, \boldsymbol{r}$ short, $\boldsymbol{m}_j$ binary vectors

$$\underbrace{[\boldsymbol{A} \mid \boldsymbol{A}_0 + \textstyle\sum_i \tau_i \boldsymbol{A}_i]}_{\boldsymbol{T_A} \text{ extends to full matrix}} \cdot \boldsymbol{v} = \boldsymbol{u} + \boldsymbol{D} \cdot \operatorname{bin}\Big(\underbrace{\boldsymbol{D}_0 \boldsymbol{r} + \textstyle\sum_j \boldsymbol{D}_j [\boldsymbol{m}_j | \boldsymbol{1} - \boldsymbol{m}_j]}_{\text{Commitment ✉}}\Big)$$

② **Packing Messages** with Variable Lengths

🔑 $= \boldsymbol{T_A}$ (Trapdoor), 🔑 $= \boldsymbol{A}_i, \boldsymbol{u}, \boldsymbol{D}, \boldsymbol{D}_j$ uniform public

$\texttt{sig} = ((\tau_i)_i, \boldsymbol{v}, \boldsymbol{r})$ with τ_i tag bits, $\boldsymbol{v}, \boldsymbol{r}$ short, $\boldsymbol{m}$ binary vector

$$[\boldsymbol{A} \mid \boldsymbol{A}_0 + \sum_i \tau_i \boldsymbol{A}_i] \cdot \boldsymbol{v} = \boldsymbol{u} + \boldsymbol{D} \cdot \text{bin}\left(\underbrace{\boldsymbol{D}_0 \boldsymbol{r} + \boldsymbol{D}_1[\boldsymbol{m} | \boldsymbol{1} - \boldsymbol{m}]}_{✉}\right)$$

$\boldsymbol{w} = \text{bin}(✉)$
- $\left[\boldsymbol{A} \mid \boldsymbol{A}_0 + \sum_i \tau_i \boldsymbol{A}_i\right] \boldsymbol{v} = \boldsymbol{u} + \boldsymbol{D}\boldsymbol{w}$
- $\text{bin-recomp}(\boldsymbol{w}) =$ ✉
- $\boldsymbol{w}$ binary

ZKP details

Before

$$\left[\boldsymbol{A} \mid \boldsymbol{A}_0 + \sum_i \tau_i \boldsymbol{A}_i\right] \cdot \boldsymbol{v} = \boldsymbol{u} + \boldsymbol{D} \cdot \text{bin}\left(\boldsymbol{D}_0 \boldsymbol{r} + \sum_j \boldsymbol{D}_j[\boldsymbol{m}_j | \boldsymbol{1} - \boldsymbol{m}_j]\right)$$

3 **New Arguments** in Security Proofs

🔑 $= \boldsymbol{T_A}$ (Trapdoor), 🔑 $= \boldsymbol{A}_i, \boldsymbol{u}, \boldsymbol{D}_j$ uniform public

$\texttt{sig} = ((\tau_i)_i, \boldsymbol{v}, \boldsymbol{r})$ with τ_i tag bits, $\boldsymbol{v}, \boldsymbol{r}$ short, $\boldsymbol{m}$ binary vector

$$[\boldsymbol{A} \mid \boldsymbol{A}_0 + \textstyle\sum_i \tau_i \boldsymbol{A}_i] \cdot \boldsymbol{v} = \boldsymbol{u} + \underbrace{\boldsymbol{D}_0 \boldsymbol{r} + \boldsymbol{D}_1 \boldsymbol{m}}_{✉}$$

Before

$$\left[\boldsymbol{A} \mid \boldsymbol{A}_0 + \textstyle\sum_i \tau_i \boldsymbol{A}_i\right] \cdot \boldsymbol{v} = \boldsymbol{u} + \boldsymbol{D} \cdot \mathrm{bin}\left(\boldsymbol{D}_0 \boldsymbol{r} + \boldsymbol{D}_1 [\boldsymbol{m} | 1 - \boldsymbol{m}]\right)$$

4 **More Compact Trapdoors** based on Gadgets

🔑 = $\boldsymbol{R}$ (Trapdoor), 🔑 = $\boldsymbol{A}, \boldsymbol{u}, \boldsymbol{D}_j$ uniform public, $\boldsymbol{G} = \boldsymbol{I} \otimes [1\ 2 \ldots 2^{k-1}]$ gadget matrix
$\mathtt{sig} = (\tau, \boldsymbol{v}, \boldsymbol{r})$ with τ tag scalar, $\boldsymbol{v}, \boldsymbol{r}$ short, $\boldsymbol{m}$ binary vector

$$[\boldsymbol{A} \mid \tau\boldsymbol{G} - \boldsymbol{A}\boldsymbol{R}] \cdot \boldsymbol{v} = \boldsymbol{u} + \underbrace{\boldsymbol{D}_0\boldsymbol{r} + \boldsymbol{D}_1\boldsymbol{m}}_{✉}$$

Before

$$\left[\boldsymbol{A} \mid \boldsymbol{A}_0 + \sum_i \tau_i \boldsymbol{A}_i\right] \cdot \boldsymbol{v} = \boldsymbol{u} + \boldsymbol{D}_0\boldsymbol{r} + \boldsymbol{D}_1\boldsymbol{m}$$

5 Compacting Commitment with Signature

= $\boldsymbol{R}$ (Trapdoor), = $\boldsymbol{A}, \boldsymbol{u}, \boldsymbol{D}_1$ uniform public, $\boldsymbol{G} = \boldsymbol{I} \otimes [1\ 2 \ldots 2^{k-1}]$ gadget matrix
sig $= (\tau, \boldsymbol{v}')$ with τ tag, $\boldsymbol{v}'$ short, $\boldsymbol{m}$ binary vector

$$[\boldsymbol{A} \mid \tau \boldsymbol{G} - \boldsymbol{A}\boldsymbol{R}]\boldsymbol{v} = \boldsymbol{u} + \underbrace{\boldsymbol{A}\boldsymbol{r} + \boldsymbol{D}_1 \boldsymbol{m}}$$

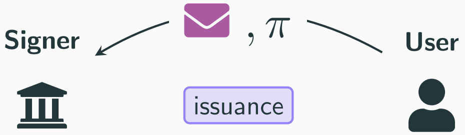
$$\Longleftrightarrow$$

$$\begin{bmatrix} \boldsymbol{A} & \mid & \tau \boldsymbol{G} - \boldsymbol{A}\boldsymbol{R} \end{bmatrix} \begin{bmatrix} \boldsymbol{v}_1' \\ \boldsymbol{v}_2 \end{bmatrix} = \boldsymbol{u} + \boldsymbol{D}_1 \boldsymbol{m} \quad \text{with} \quad \boldsymbol{v}_1' = \boldsymbol{v}_1 - \boldsymbol{r}$$

Before

$$\begin{bmatrix} \boldsymbol{A} & \mid & \tau \boldsymbol{G} - \boldsymbol{A}\boldsymbol{R} \end{bmatrix} \cdot \boldsymbol{v} = \boldsymbol{u} + \boldsymbol{D}_0 \boldsymbol{r} + \boldsymbol{D}_1 \boldsymbol{m}$$

Application to Anonymous Credentials: The Protocols



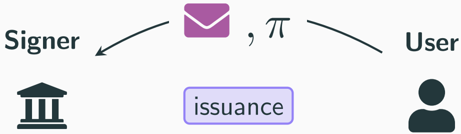
❶ ✉ $= \boldsymbol{A}\boldsymbol{r} + \boldsymbol{D}\boldsymbol{m}$

❷ $\pi = \mathrm{Proof}($✉$, \boldsymbol{r}, \boldsymbol{m})$

[LNP22][6] (lin.)

[6] V. Lyubashevsky, N. K. Nguyen, M. Plançon. Lattice-Based Zero-Knowledge Proofs and Applications: Shorter, Simpler, and More General. Crypto 2022.

❸ Verif(✉, π)



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✉ , π

Signer ← **User**

❸ Verif(✉, π)

❹ $\tau \in \mathcal{T}$

issuance

❶ ✉ $= \boldsymbol{A}\boldsymbol{r} + \boldsymbol{D}\boldsymbol{m}$

❷ $\pi =$ Proof(✉, $\boldsymbol{r}$, $\boldsymbol{m}$)

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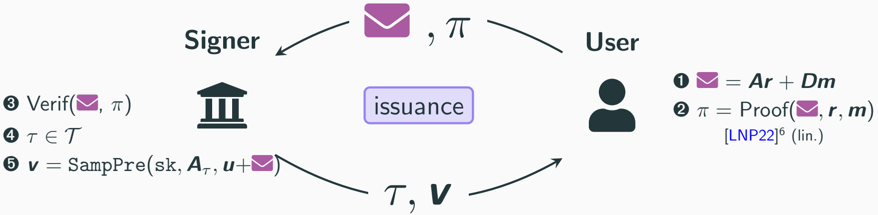
Credential Issuance and Showing



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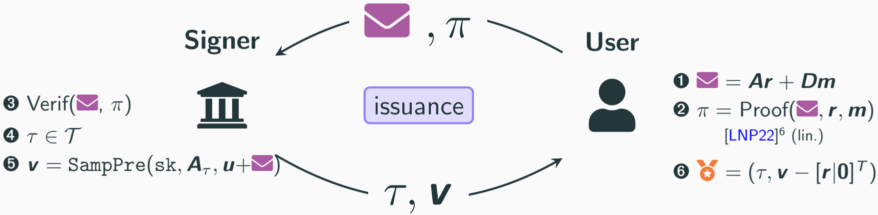

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Credential Issuance and Showing

Signer ← ✉, π ← **User**

issuance

❸ Verif(✉, π)

❹ $\tau \in \mathcal{T}$

❺ $\boldsymbol{v} = \texttt{SampPre}(\texttt{sk}, \boldsymbol{A}_\tau, \boldsymbol{u} +$ ✉$)$

❶ ✉ $= \boldsymbol{A}\boldsymbol{r} + \boldsymbol{D}\boldsymbol{m}$

❷ $\pi =$ Proof(✉, $\boldsymbol{r}, \boldsymbol{m}$)
[LNP22][6] (lin.)

Signer → $\mathcal{T}, \boldsymbol{V}$ → **User**

❻ 🏅 $= (\tau, \boldsymbol{v} - [\boldsymbol{r}|0]^T)$

User → π → **Verifiers**

showing

❶ $\pi =$ Proof(🏅, $\boldsymbol{m}$)
[LNP22] (quad.)

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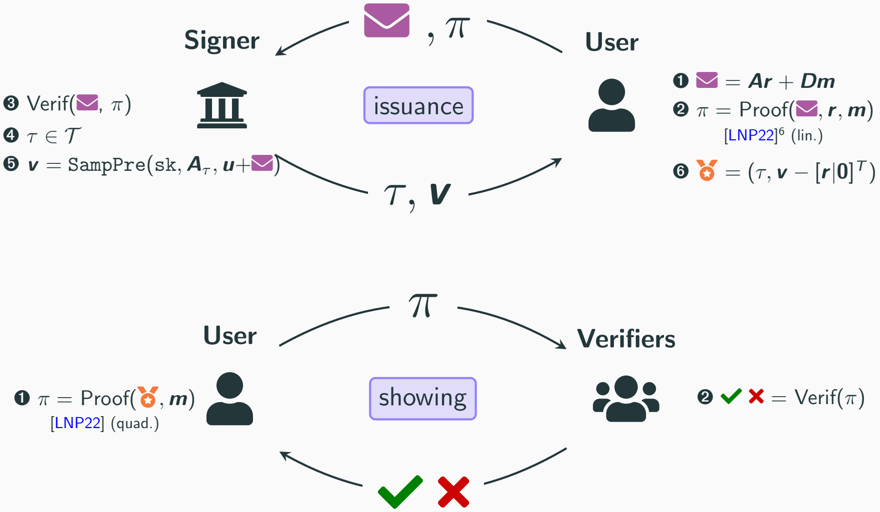

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Security of Anonymous Credentials

- **Anonymity**:
  - *Issuance.* No leakage of the secret key, nor concealed attributes
    - ✓ Hiding commitment, and Zero-Knowledge
  - *Showing.* No leakage of the credential, secret, concealed attributes
    - ✓ Zero-Knowledge

- **Unforgeability**: Prevent three types of forgeries.
  - *Impersonation.* Forgery using an honest user's secret key
    - ✓ Reduction to Module-SIS with matrix $\boldsymbol{D}_s$
  - *Malicious Prover.* Tricks verifiers in the zero-knowledge argument
    - ✓ Soundness of the proof system
  - *Signature Forgery.* Forges a valid credential on fresh attributes/key
    - ✓ EUF-CMA security of our signature

Conclusion

Wrapping Up

Our contribution (https://ia.cr/2022/509)

- A (more) practical **signature with efficient protocols**, under standard or structured **lattice assumptions**.
- **Orders of magnitude more efficient** than [LLM+16].
- **Fix** of the approximate ZK proof system of [YAZ+19].
- First **lattice-based anonymous credentials**.

Related Work

	Assumptions	Interactive Assumption	|cred|
[LLM+16]	SIS	No	670 MB (appr. proof)
Ours	MSIS/MLWE	No	730 KB
[BLNS23]	NTRU-ISIS$_f$	No	243 KB
	Int-NTRU-ISIS$_f$	Yes	62 KB

Thank you for your attention!

Questions?

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