Lattice Signature with Efficient Protocols, Application to Anonymous Credentials

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Corentin Jeudy^{1,2}, Adeline Roux-Langlois³, Olivier Sanders¹

Reminder: Digital Signatures



Signature with Efficient Protocols (SEP)



An Interesting Versatility

Many concrete privacy-enhancing applications.

Anonymous Credentials Systems: requires the ability to

- ✓ sign committed messages
- \checkmark prove possession of a message-signature pair in ZK

Group Signatures: requires to add a verifiable encryption of the user identity

Blind Signatures: requires the ability to

- sign committed messages
- \checkmark prove possession of a signature on a public message in ZK

E-Cash Systems

etc.

Real industrial impact: EPID and DAA deployed in billions of devices (TPM, Intel SGX enclaves)

Very efficient instantiations of SEPs in the classical setting.

[CL02]¹ Based on the Strong-RSA assumption. [CL04]²[BB08]³[PS16]⁴ Based on pairings in bilinear groups.

[BB08][PS16] are constant-size. Very efficient group signatures, anonymous credentials, etc.

Group Signature based on SEP: 0.16 KB

¹J. Camenisch, A. Lysyanskaya. A signature scheme with efficient protocols. SCN 2002.

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Those are vulnerable to quantum computing. How about **post-quantum** solutions?

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NIST **PQC standardization process** launched in 2016. First round of standardized algorithms announced in July 2022:

Signature Schemes					
Crystals-Dilithium Falcon	lattice-based but in the ROM 4				
SPHINCS+					

NIST called for new signatures without lattices starting in June 2023.



Only **standard signatures** are considered, focus on efficiency rather than functionality. How about signatures for privacy-enhancing protocols?

Only one proposal of post-quantum signature with efficient protocols: [LLM+16]⁵ Proof of concept based on standard lattices.

		pk	sk	sig	$ \pi $
[LLM+16]	Exact Proof	3 TB	15 GB	9 MB	10 GB
	Appr. Proof	7 TB	37 GB	14 MB	670 MB

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Simpler, more compact, more efficient construction on standard lattices, and extension to ideal and module lattices.

		pk	sk	sig	$ \pi $
Ours	Exact Proof	8 MB	9 MB	270 KB	640 KB

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Today

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Short Integer Solution and Trapdoors

(Module-)SIS_{$m,d,q,\beta_2,\beta_\infty$}

Given $\mathbf{A} \leftrightarrow U((R/qR)^{d \times m})$, find a **non-zero** $\mathbf{x} \in R^m$ such that $\mathbf{A}\mathbf{x} = \mathbf{0} \mod qR$, $0 < \|\mathbf{x}\|_2 \le \beta_2$ and $0 < \|\mathbf{x}\|_{\infty} \le \beta_{\infty}$. $R = \mathbb{Z}$ (Standard-)SIS $R = \mathbb{Z}[x]/\langle x^n + 1 \rangle$ with $n = 2^k$ Module-SIS

Trapdoor on **A**: piece of information used to sample short vector **x** such that $Ax = u \mod qR$ for any syndrome u



Lattice Signature With Efficient Protocols

Original Construction from [LLM⁺16]

 $\begin{aligned} & \texttt{sk} = \textit{T}_{\textit{A}} (\mathsf{Trapdoor}), \textit{A}_i, \textit{u}, \textit{D}, \textit{D}_j \text{ uniform public} \\ & \texttt{sig} = ((\tau_i)_i, \textit{v}, \textit{r}) \text{ with } \tau_i \text{ tag bits, } \textit{v}, \textit{r} \text{ short, } m_j \text{ binary vectors} \end{aligned}$

$$\begin{bmatrix} \mathbf{A} & | & \mathbf{A}_0 + \sum_j \boldsymbol{\tau}_j \mathbf{A}_j \end{bmatrix} \cdot \mathbf{v} = \mathbf{u} + \mathbf{D} \cdot \operatorname{bin} \left(\mathbf{D}_0 \mathbf{r} + \sum_j \mathbf{D}_j [\mathbf{m}_j | \mathbf{1} - \mathbf{m}_j] \right)$$



Packing Messages with Variable Lengths

 $sk = T_A$ (Trapdoor), A_i, u, D, D_j uniform public $sig = ((\tau_i)_i, v, r)$ with τ_i tag bits, v, r short, m binary vector

$$\begin{bmatrix} \boldsymbol{A} & | & \boldsymbol{A}_0 + \sum_i \boldsymbol{\tau}_i \boldsymbol{A}_i \end{bmatrix} \boldsymbol{v} = \boldsymbol{u} + \boldsymbol{D} \cdot \operatorname{bin} \left(\boldsymbol{D}_0 \boldsymbol{r} + \boldsymbol{D}_1 [\boldsymbol{m} | \boldsymbol{1} - \boldsymbol{m}] \right)$$

$$\begin{pmatrix} w = bin (D_0 r + D_1[m|1 - m]) \\ \bullet [A | A_0 + \sum_i \tau_i A_i] v = u + Dw \\ \bullet bin-recomp(w) = D_0 r + D_1[m|1 - m] \\ \bullet w binary \end{pmatrix} ZKP details (tl;dr)$$

Before

$$\begin{bmatrix} \mathbf{A} & | & \mathbf{A}_0 + \sum_i \tau_i \mathbf{A}_i \end{bmatrix} \cdot \mathbf{v} = \mathbf{u} + \mathbf{D} \cdot \operatorname{bin} \left(\mathbf{D}_0 \mathbf{r} + \sum_j \mathbf{D}_j [\mathbf{m}_j | \mathbf{1} - \mathbf{m}_j] \right)$$

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New Arguments in Security Proofs

 $sk = T_A$ (Trapdoor), A_i, u, D_j uniform public $sig = ((\tau_i)_i, v, r)$ with τ_i tag bits, v, r short, m binary vector

$$\begin{bmatrix} \mathbf{A} \mid \mathbf{A}_0 + \sum_i \boldsymbol{\tau}_i \mathbf{A}_i \end{bmatrix} \mathbf{v} = \mathbf{u} + \mathbf{D}_0 \mathbf{r} + \mathbf{D}_1 \mathbf{m}$$

Before

$$\begin{bmatrix} \mathbf{A} & | & \mathbf{A}_0 + \sum_i \tau_i \mathbf{A}_i \end{bmatrix} \cdot \mathbf{v} = \mathbf{u} + \mathbf{D} \cdot \operatorname{bin} \left(\mathbf{D}_0 \mathbf{r} + \mathbf{D}_1 [\mathbf{m} | \mathbf{1} - \mathbf{m}] \right)$$

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More Compact Trapdoors based on Gadgets

sk = R (Trapdoor), A, u, D_j uniform public, $G = I \otimes [1 \ 2 \dots 2^{k-1}]$ gadget matrix $sig = (\tau, v, r)$ with τ tag, v, r short, m binary vector

$$egin{bmatrix} \mathsf{A} & \mid & oldsymbol{ au} \mathbf{G} - oldsymbol{A} \mathbf{R} \end{bmatrix} \mathbf{v} = oldsymbol{u} + oldsymbol{D}_0 oldsymbol{r} + oldsymbol{D}_1 oldsymbol{m}$$

Before

$$\begin{bmatrix} \mathbf{A} & | & \mathbf{A}_0 + \sum_i \tau_i \mathbf{A}_i \end{bmatrix} \cdot \mathbf{v} = \mathbf{u} + \mathbf{D}_0 \mathbf{r} + \mathbf{D}_1 \mathbf{m}$$

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Compacting Commitment with Signature

sk = R (Trapdoor), A, u, D_1 uniform public, $G = I \otimes [1 \ 2 \dots 2^{k-1}]$ gadget matrix sig = (τ, v') with τ tag, v' short, m binary vector

$$\begin{bmatrix} A & | \quad \tau G - AR \end{bmatrix} \mathbf{v} = \mathbf{u} + A\mathbf{r} + D_1 \mathbf{m}$$

$$\iff$$

$$A & | \quad \tau G - AR \end{bmatrix} \begin{bmatrix} \mathbf{v}_1' \\ \mathbf{v}_2 \end{bmatrix} = \mathbf{u} + D_1 \mathbf{m} \quad \text{with} \quad \mathbf{v}_1' = \mathbf{v}_1 - \mathbf{r}$$

Before
$$\begin{bmatrix} A & | & \tau G - AR \end{bmatrix} \mathbf{v} = \mathbf{u} + D_0 \mathbf{r} + D_1 \mathbf{m}$$

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Performance

		pk	sk	sig	$ \pi $
[LLM+16]	Exact Proof	3 TB	15 GB	9 MB	10 GB
	Appr. Proof	7 TB	37 GB	14 MB	670 MB
Ours (\mathbb{Z}_q)	Exact Proof	1 GB	1 GB	250 KB	300 MB
	Appr. Proof	3 GB	2 GB	400 KB	18 MB

Adaptable to an algebraic setting for more efficiency, unlike [LLM+16].



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Application to Anonymous Credentials: The Protocols

What Are Those Efficient Protocols?



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Issuance of Credentials



We had:



Issuance of Credentials



Security proof subtleties

Signer must contribute to the commitment randomness:

User has a key pair $(sk, pk) = (s, D_s s)$, and must include s in its attributes, and prove knowledge of its key pair.

$$\mathbf{0} = \mathbf{A}\mathbf{r} + \mathbf{D}\mathbf{m} + \mathbf{D}_s \mathbf{s}$$

 $\boldsymbol{2} \pi = \mathsf{Proof}(\boldsymbol{\boxtimes}, \boldsymbol{r}, \boldsymbol{m}; \mathsf{pk}, \boldsymbol{s})$

Showing of Credentials



Showing of Credentials



 $^{^{6}}V.$ Lyubashevsky, N. K. Nguyen, M. Plançon. Lattice-Based Zero-Knowledge Proofs and Applications: Shorter, Simpler, and More General. Crypto 2022.

Security of Anonymous Credentials

Anonymity:

Issuance. No leakage of the secret key, nor concealed attributes
 Hiding commitment, and Zero-Knowledge
 Showing. No leakage of the credential, secret, concealed attributes

Zero-Knowledge

Unforgeability: Prevent three types of forgeries.

Impersonation. Forgery using an honest user's secret key

✓ Reduction to SIS with matrix D_s

Malicious Prover. Tricks verifiers in the zero-knowledge argument

 Soundness of the proof system
 Signature Forgery. Forges a valid credential on fresh attributes/key



Wrapping Up

Our contribution (https://ia.cr/2022/509)

- A practical signature with efficient protocols, under standard or structured lattice assumptions.
- Several orders of magnitude more efficient than the only lattice construction.
- **Fix** of the approximate ZK proof system of [YAZ⁺19].
 - First lattice-based anonymous credentials.

Future Work

Anonymous credentials require potentially very large messages. Proving knowledge of all the hidden attributes can be costly.

Replace regular commitment with a **vector commitment** to allow specific short openings.

Several optimizations on track.

Sneak Peak

All the results were in the statistical setting

		pk	sk	sig	$ \pi $
[LLM ⁺ 16]	Exact	3 TB	15 GB	9 MB	10 GB
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Ours (R_q)	Exact	8 MB	9 MB	270 KB	640 KB

Better computationally with other tricks (ongoing work, rough estimates).

		pk	sk	sig	$ \pi $
New	Exact	55 KB	10 KB	12 KB	87 KB
		×149	×921	×22.5	×7.5

Thank you for your attention!

Questions?

References

D. Boneh and X. Boyen.

Short Signatures Without Random Oracles and the SDH Assumption in Bilinear Groups.

J. Cryptol., 2008.

J. Camenisch and A. Lysyanskaya.

A Signature Scheme with Efficient Protocols. In <u>SCN</u>, 2002.

- J. Camenisch and A. Lysyanskaya. Signature Schemes and Anonymous Credentials from Bilinear Maps.
 - In <u>CRYPTO</u>, 2004.

References

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 C. Jeudy, A. Roux-Langlois, and O. Sanders.
 Lattice Signature with Efficient Protocols, Application to Anonymous Credentials.

IACR Cryptol. ePrint Arch., page 509, 2022.

- Q. Lai, F.-H. Liu, A. Lysyanskaya, and Z. Wang.
 Lattice-based Commit-Transferrable Signatures and Applications to Anonymous Credentials.
 IACR Cryptol. ePrint Arch., page 766, 2023.

B. Libert, S. Ling, F. Mouhartem, K. Nguyen, and H. Wang.
Signature Schemes with Efficient Protocols and Dynamic
Group Signatures from Lattice Assumptions.
In ASIACRYPT, 2016.

References iii

 V. Lyubashevsky, N. K. Nguyen, and M. Plançon.
 Lattice-Based Zero-Knowledge Proofs and Applications: Shorter, Simpler, and More General.
 CRYPTO, 2022.

D. Pointcheval and O. Sanders.
 Short Randomizable Signatures.
 In CT-RSA, 2016.



G. Policharla, B. Westerbaan, A. Faz-Hernández, and C. A. Wood. Post-Quantum Privacy Pass via Post-Quantum Anonymous Credentials.

IACR Cryptol. ePrint Arch., page 414, 2023.





R. Yang, M. H. Au, Z. Zhang, Q. Xu, Z. Yu, and W. Whyte. Efficient Lattice-Based Zero-Knowledge Arguments with Standard Soundness: Construction and Applications. In <u>CRYPTO</u>, 2019.

How About Security?



It is **EUF-CMA secure** based on **SIS/M-SIS**, with polynomial loss for adaptive security.

Sketch:

• If the forge must use an unseen tag, we make a guess τ^+ and generate $\boldsymbol{B} = \boldsymbol{A}\boldsymbol{R} + \tau^+\boldsymbol{G}$ (instead of $\boldsymbol{A}\boldsymbol{R}$), $\boldsymbol{D} = \boldsymbol{A}\boldsymbol{U}$ for short random \boldsymbol{U} . If the forgery (τ^*, \mathbf{v}^*) on \boldsymbol{m}^* verifies and satisfies $\tau^* = \tau^+$, then

$$[\mathbf{A}|\tau^*\mathbf{G}-\mathbf{B}]\mathbf{v}^*=\mathbf{u}+\mathbf{D}\mathbf{m}^* \Longleftrightarrow [\mathbf{A}|\mathbf{u}]\begin{bmatrix} [\mathbf{I}|-\mathbf{R}]\mathbf{v}^*-\mathbf{U}\mathbf{m}^*\\-1\end{bmatrix}=\mathbf{0}.$$

② If a tag is re-used, we guess which one τ_i . We generate $\boldsymbol{B} = \boldsymbol{A}\boldsymbol{R} + \tau_i \boldsymbol{G}$, $\boldsymbol{D} = \boldsymbol{A}\boldsymbol{U}$ and $\boldsymbol{u} = [\boldsymbol{A}| - \boldsymbol{A}\boldsymbol{R}](\boldsymbol{v} - [\boldsymbol{r}_0^T|\boldsymbol{0}^T]^T)$. To answer the *i*-th query, we return $(\tau_i, \boldsymbol{v}_i = \boldsymbol{v} - [(\boldsymbol{r}_0 - \boldsymbol{U}\boldsymbol{m}_i)^T|\boldsymbol{0}^T]^T)$. If the forgery $(\tau^*, \boldsymbol{v}^*)$ on \boldsymbol{m}^* verifies and satisfies $\tau^* = \tau_i$, then

$$A\left([I|-R](v^*-v_i)-U(m^*-m_i)\right)=0.$$

Recent Related Works

	Assumptions	Security	Interactive Assumption	pk	cred
Ours [JRS22] ⁷	MSIS/MLWE	Adaptive	No	10 MB	730 KB
[?] ⁸	$\frac{NTRU-ISIS_f}{Int-NTRU-ISIS_f}$	Adaptive Adaptive	No Yes	17 KB 3.5 KB	243 KB 62 KB
[PWFW23]] ⁹ Sym/ST-MSIS	Adaptive	No?	?	180 KB?
[LLLW23]	⁰ MSIS/MLWE MSIS/MLWE	Selective Adaptive	No No	238 KB 40 MB	232 KB 3 MB

⁷C. Jeudy, A. Roux-Langlois, O. Sanders. Lattice Signature with Efficient Protocols, Application to Anonymous Credentials. ePrint 2022/509.

⁸ J. Bootle, V. Lyubashevsky, N. K. Ngyuen, A. Sorniotti. A Framework for Practical Anonymous Credentials from Lattices. ePrint 2023/560.

⁹G.-V. Policharla, B. Westerbaan, A. Fas-Hernández, C. A. Wood. Post-Quantum Privacy Pass via Post-Quantum Anonymous Credentials. ePrint 2023/414.

¹⁰Q. Lai, F.-H. Liu, A. Lysyanskaya, Z. Wang. Lattice-based Commit-Transferrable Signatures and Applications to Anonymous Credentials. ePrint 2023/766.